ROLL NO. ___________________

Code: AC65/AC116 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current & New Scheme)

Time: 3 Hours **JUNE 2017** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Let A and B be any two arbitrary events then which one of the following is true?
(A) $P(A \cap B) = P(A) \cdot P(B)$ **(B)** $P(A \cup B) = P(A) + P(B)$
(C) $P(AB) = P(A \cap B) \cdot P(B)$ **(D)** $P(A \cup B) \geq P(A) + P(B)$

b. Probability that two randomly selected cards from a set of two red and two black cards are of same color is?
(A) 1/2 **(B)** 1/3
(C) 2/3 **(D)** None of these

c. What is the Cartesian Product of $A = \{1,2\}$ and $B = \{a,b\}$
(A) $\{(1,a),(1,b),(2,a),(b,b)\}$ **(B)** $\{(1,1),(2,2),(a,a),(b,b)\}$
(C) $\{(1,a),(2,a),(1,b),(2,b)\}$ **(D)** $\{(1,1),(a,a),(2,a),(1,b)\}$

d. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers, then F_4 *is*
(A) 3 **(B)** 4
(C) 5 **(D)** 6

e. On the set $A = \{1,2,3,4,5,6\}$, the relation R is defined as follows:
$R = \{(1,2),(1,6),(2,3),(3,3),(4,1),(4,3),(4,5),(6,4)\}$. Then the length of the path from 6 to 2 is
(A) 2 **(B)** 3
(C) 4 **(D)** 5

f. Relation defined on a partially order is not
(A) Symmetric **(B)** Transitive
(C) Relexive **(D)** Anti-symmetric

g. The value of $\left\lfloor \frac{1}{2} \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$ is
(A) 1 **(B)** 2
(C) 3 **(D)** 0.5

h. If A is finite set with $|A| = 4$, how many binary operations can be defined on A such that they are commutative?
(A) 4^2 **(B)** 4^4
(C) 4^{10} **(D)** 4^{16}

i. The weight of the word 10100 is
(A) 2 **(B)** 3
(C) 4 **(D)** 5

j. Every ____________ is an integral domain
(A) group **(B)** ring
(C) abelian group **(D)** commutative division ring

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A survey of 500 television viewers of a sports channel produced the following information: 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch football and cricket, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three kinds of games. How many viewers watch exactly one sport? **(8)**

b. If U={1,2,3,4,5,6,7,8,9}, A={1,2,4,6,8} and B={2,4,5,9}. Find $\overline{A}\cup\overline{B}, \overline{A}\cap\overline{B}$ and $A\Delta B$. **(4)**

c. A problem is given to four students A,B,C,D whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem is solved. **(4)**

Q.3 a. Prove that, for any three propositions p, q, r,
$[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$. **(8)**

b. Examine whether $[(p \vee q) \to r] \leftrightarrow [\neg r \to \neg(p \vee q)]$ is a tautology **(8)**

Q.4 a. The generator matrix for an encoding function $E : Z_2^3 \to Z_2^6$ is given by
$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$ Find the code words assigned to 110 and 010.
Obtain the associated parity-check matrix. Hence decode the received words: 110110, 111101. Show that the decoding of 111111 is not possible by using H. **(8)**

b. Prove that the set Z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a commutative ring with unity. **(8)**

Q.5 a. Prove by mathematical induction that, for any positive integer n, the number $11^{n+2} + 12^{2n+1}$ is divisible by 133. **(8)**

b. Let A={1,3,5}, B={2,3} and C={4,6}. Find $(A \cup B) \times C$, $A \cup (B \times C)$, $(A \times B) \cup (B \times C)$ and $(A \times B) \cap (B \times C)$. **(8)**

Q.6 a. Find whether the following argument is valid:
No engineering student of first or second semester studies logic.
Anil is an engineering student who studies logic.
$\therefore$ Anil is not in second semester. **(8)**

b. Give a direct proof, an indirect proof and proof by contradiction for the statement "If n is an odd integer, then n+11 is an even integer" **(8)**

Q.7 a. For a fixed integer n > 1, prove that the relation "congruent modulo n" is an equivalence relation on the set of all integers, **Z**. **(8)**

b. Let A = {a,b,c}, and R and S be relations on A whose matrices are as given below:

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \; M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find the composite relations $R \circ S$, $S \circ R$, $R \circ R$, $S \circ S$ and their matrices. **(8)**

Q.8 a. Define one-to-one, onto functions. Give an example of a function which is
(i) one-to-one but not onto (ii) onto but not one-to-one
(iii) neither onto nor one-to-one (iv) both one-to-one and onto **(8)**

b. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be permutation of the set A ={1,2,3,4,5,6}. Write p as a product of disjoint cycles. Compute p^{-1} and p^3. Find the smallest integer k such that $p^k = I_A$. **(8)**

Q.9 a. If G is a finite group and H is a subgroup of G, prove that the order of H divides the order of G. **(8)**

b. Prove that the group $(Z_6, +)$ is cyclic. Find all its generators. Also prove that $(Z_6, +)$ and $(U_9, .)$ are isomorphic. **(8)**