

Time: 3 Hours

JUNE 2013

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. The value of $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right)^{\tan x}$ is:

- (A) -1 (B) 0
(C) 1 (D) 2

b. The value of $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$ is:

- (A) $\frac{3\pi}{512}$ (B) $\frac{\pi}{128}$
(C) $\frac{-\pi}{128}$ (D) $\frac{-3\pi}{512}$

c. The multiplicative inverse of $3-4i$ is:

- (A) $\frac{4}{25} + \frac{3}{25}i$ (B) $\frac{3}{25} - \frac{4}{25}i$
(C) $\frac{3}{25} + \frac{4}{25}i$ (D) $\frac{4}{25} - \frac{3}{25}i$

d. The area of the parallelogram formed by the vectors $\vec{a} = 3\hat{i} + 2\hat{j}$, $\vec{b} = 2\hat{j} - 4\hat{k}$ is:

- (A) $4\sqrt{61}$ sq units (B) $2\sqrt{61}$ sq units
(C) $3\sqrt{61}$ sq units (D) $\sqrt{61}$ sq units

e. The value of λ such that the vectors $\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 5\hat{i} - 9\hat{j} + 2\hat{k}$ are perpendicular to each other is:

- | | |
|---------------------|--------------------|
| (A) $\frac{5}{16}$ | (B) $\frac{5}{24}$ |
| (C) $\frac{-5}{16}$ | (D) $\frac{16}{5}$ |

f. If $\frac{d^2y}{dx^2} - y = 2 + 3x$, then C.F. is:

- | | |
|------------------------------------|--------------------------------|
| (A) $C_1e^x + C_2e^{-x}$ | (B) $C_1 \cos x + C_2 \sin 3x$ |
| (C) $e^x(C_1 \cos x + C_2 \sin x)$ | (D) $C_1e^x + C_2e^{2x}$ |

g. If $f(x) = x$, as a Fourier series in the interval $[-\pi, \pi]$ then the value of a_0 is:

- | | |
|--------|-------|
| (A) -1 | (B) 2 |
| (C) 0 | (D) 3 |

h. Value of $L[5\sin 2t - 3\cos 2t]$ is:

- | | |
|----------------------------------|----------------------------------|
| (A) $\frac{3s-10}{s^2+4}, s > 0$ | (B) $\frac{10-3s}{s^2+4}, s > 0$ |
| (C) $\frac{5s+6}{s^2+4}, s > 0$ | (D) $\frac{6-5s}{s^2+4}, s > 0$ |

i. Value of $L[e^{3t} \sin 4t]$ is:

- | | |
|-------------------------------|-------------------------------|
| (A) $\frac{4}{s^2 - 6s + 25}$ | (B) $\frac{4}{s^2 + 6s + 25}$ |
| (C) $\frac{4}{s^2 - 6s + 9}$ | (D) $\frac{4}{s^2 + 6s + 9}$ |

j. The value of $L^{-1}\left[\frac{4s+15}{16s^2-25}\right]$ is:

- | | |
|---|---|
| (A) $\frac{1}{4} \cosh\left(\frac{5}{4}t\right) + \sinh\left(\frac{4}{5}t\right)$ | (B) $\cosh\left(\frac{4}{5}t\right) + \sinh\left(\frac{4}{5}t\right)$ |
| (C) $\frac{1}{4} \cosh\left(\frac{5}{4}t\right) + \frac{3}{4} \sinh\left(\frac{5}{4}t\right)$ | (D) $\cosh\left(\frac{5}{4}t\right) - \frac{3}{4} \sinh\left(\frac{5}{4}t\right)$ |

Answer any FIVE Questions out of EIGHT Questions.

Each question carries 16 marks.

Q.2 a. Evaluate $\lim_{x \rightarrow 0} \frac{\log \sin 2x}{\log \sin x}$ (8)

- b. Expand $\cos x$ in powers of $\left(x - \frac{\pi}{4}\right)$ upto 4 terms (using Taylor's Expansion). (8)
- Q.3** a. Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$. (8)
- b. Find the volume generated by revolving the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ about the x-axis. (8)
- Q.4** a. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$. (8)
- b. Prove that
- $$(1+i)^n + (1-i)^n = 2^{(n/2)+1} \cos\left(\frac{n\pi}{4}\right) \quad (8)$$
- Q.5** a. What is the unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ & $3\hat{i} + 4\hat{j} - \hat{k}$? Calculate the sine of the angle between these two vectors. (8)
- b. A force is represented in magnitude and direction by the line joining the point A(1,-2,4) to the point B(5,2,3). Find its moment about the point (-2,3,5). (8)
- Q.6** a. Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$ (8)
- b. Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 2x^2$, given that $y(0) = 0$ and $y'(0) = 0$. (8)
- Q.7** a. Obtain a Fourier series representation for $f(x)$ where
- $$f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 < x < 2\pi. \quad (8)$$
- b. Find the Fourier sine series which represents $f(x) = \pi - x$ in the interval $(0, \pi)$ (8)
- Q.8** a. Find the Laplace transform of $t^2 \cos at$ (8)
- b. Find Laplace transform of $\frac{1 - e^{2t}}{t}$ (8)
- Q.9** a. Find $L^{-1} \left\{ \frac{3s + 9}{(s^2 + 2s + 10)} \right\}$ (8)
- b. Use convolution theorem to find $L^{-1} \left\{ \frac{1}{(s^2 - s - 2)} \right\}$ (8)