Code: DE51/DC51   Subject: ENGINEERING MATHEMATICS - I

DipIETE – ET/CS

Time: 3 Hours   **JUNE 2013**   **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1   Choose the correct or the best alternative in the following: **(2×10)**

a. $\text{Lim}_{x \to 0} \dfrac{\sin ax}{\sin bx}$ is :

   (A) $\dfrac{1}{b}$ **(B)** a

   (C) 0   **(D)** $\dfrac{a}{b}$

b. The ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7)

   (A) 3:4 externally   **(B)** 3:4 internally
   (C) 4:3 internally   **(D)** 4:3 externally

c. $\int \dfrac{\sin 4x}{\sin x} dx$ is

   (A) $2\left[\dfrac{\sin 3x}{3} + \sin x\right] + c$ **(B)** $\dfrac{\sin 3x}{3} + \sin x + c$

   (C) $2\left[\dfrac{\sin 3x}{3} - \sin x\right] + c$ **(D)** $\dfrac{\sin 3x}{3} - \sin x + c$

d. If $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$, then the value of Δ is

   (A) 3   **(B)** 1
   (C) 0   **(D)** 4

e. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -2 \\ 1 & -5 \\ 4 & 3 \end{bmatrix}$ and A+B-X=0, then the value of X is

(A) $\begin{bmatrix} 2 & 3 \\ -1 & 0 \\ 4 & 9 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 4 \\ -1 & 4 \\ 9 & 8 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 0 \\ -1 & 4 \\ 9 & 9 \end{bmatrix}$

(D) $\begin{bmatrix} -2 & 0 \\ 4 & -1 \\ 9 & 9 \end{bmatrix}$

f. The order and degree of differential equation $y = \dfrac{dy}{dx} + \dfrac{c}{\dfrac{dy}{dx}}$ is

(A) O = 1, D = 1   (B) O = 1, D = 2
(C) O = 2, D = 1   (D) O = 2, D = 2

g. The middle term in the expansion of $\left(x - \dfrac{1}{2y} \right)^{10}$ is

(A) $-\dfrac{63x^5}{8y^5}$

(B) $-\dfrac{63y^5}{8x^5}$

(C) $\dfrac{8x^5}{63y^5}$

(D) $\dfrac{8y^5}{63x^5}$

h. If $\cot\alpha\cot\beta = 2$, then the value of $\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$ is

(A) $-\dfrac{1}{3}$

(B) $\dfrac{1}{3}$

(C) 2

(D) $\dfrac{1}{2}$

i. The distance between the pair of points A(2, 5), B(-3, 7) is

(A) $\sqrt{30}$ (B) $\sqrt{28}$
(C) $\sqrt{27}$ (D) $\sqrt{29}$

j. If $y = \log\sin x$, then $\dfrac{dy}{dx}$ is

(A) tan x   (B) -tan x
(C) cot x   (D) -cot x

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $\sin y = x\sin(a+y)$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$. **(8)**

b. Prove that the straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point where the curve crosses the axis of y. **(8)**

Q.3 a. Evaluate $\displaystyle\int \dfrac{1}{\sqrt{x(1-2x)}}\,dx$ **(8)**

b. Evaluate $\displaystyle\int_{0}^{\pi/2} \dfrac{\cos\theta}{(1+\sin\theta)(2+\sin\theta)}\,d\theta$ **(8)**

Q.4 a. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ **(8)**

b. Apply Cramer's rule to solve the following system of linear equations
$x + y + z = -1$
$x + 2y + 3z = -4$ **(8)**
$x + 3y + 4z = -6$

Q.5 a. Solve $x\dfrac{dy}{dx} + \cot y = 0$, given that $y = \dfrac{\pi}{4}$ when $x = \sqrt{2}$ **(8)**

b. Solve $\dfrac{dy}{dx} + y\sec x = \tan x$ **(8)**

Q.6 a. Prove that $\cos^2 A + \cos^2(A+120°) + \cos^2(A-120°) = \dfrac{3}{2}$ **(8)**

b. If $A + B + C = \pi$, prove that $\cot\dfrac{A}{2} + \cot\dfrac{B}{2} + \cot\dfrac{C}{2} = \cot\dfrac{A}{2}\cot\dfrac{B}{2}\cot\dfrac{C}{2}$ **(8)**

Q.7 a. Find the term independent of 'x' in the expansion of $\left(\dfrac{3x^2}{2} - \dfrac{1}{3x}\right)^9$ **(8)**

b. If the 5th term of a G.P. is 16 and the 10th term is $\dfrac{1}{2}$, find the G.P. Also find its 15th term. **(8)**

Q.8 a. A line passes through (3, 4) and the sum of its intercepts on the axis is 14, find the equation of the line. **(8)**

   b. Find the distance between the lines $9x + 40y - 20 = 0$ and $9x + 40y + 21 = 0$ **(8)**

Q.9 a. Find the equation of the circle whose centre is the point $(1,-2)$ and which passes through the centre of the circle $x^2 + y^2 + 2y - 3 = 0$ **(8)**

   b. Find the equation of the parabola whose focus is $(4,-3)$ and whose vertex is $(4,1)$. **(8)**