ROLL NO. ________________

Code: AE73 Subject: INFORMATION THEORY & CODING

AMIETE – ET

Time: 3 Hours **JUNE 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. A probability density function is given by $P(x) = ke^{-x^2/2}, -\infty < x < \infty$. The value of k is

(A) $\frac{1}{\sqrt{2\pi}}$ **(B)** $\sqrt{\frac{2}{\pi}}$

(C) $\frac{1}{2\sqrt{\pi}}$ **(D)** $\frac{1}{\pi\sqrt{2}}$

b. The spectral density of real valued random process has

(A) an even symmetry **(B)** an odd symmetry

(C) a conjugate symmetry **(D)** no symmetry

c. The imaginary channel rejection in a superheterodyne receiver comes from

(A) IF stages only **(B)** RF stages only

(C) Detector and RF stages **(D)** Detector, RF & IF stages

d. If Y and Z are random variables obtained by sampling X(t) at t = 2 and t = 4 respectively and let W = Y – Z. The variance of W is

(A) 13.36 **(B)** 9.36

(C) 2.64 **(D)** 8.00

e. Auto correlation function of a random process is

(A) $R(t_1, t_2) = E(XY) = \iint xy\, p(x, y)\, dxdy$

(B) $E(XY) = \iint x^2 y^2 dxdy$

(C) $R(t_1, t_2) = \iint x^2 y^2 dxdy$

(D) None of these

f. The differential entropy of N_k as

(A) $h(N_k) = \frac{1}{2}\log_2\left[2\pi e\left(P+\sigma^2\right)\right]$ **(B)** $h(N_k) = \frac{1}{2}\log_2\left(2\pi e\sigma^2\right)$

(C) Both **(A)** & **(B)** **(D)** None of these

g. In a SEC Hamming code, the number of message bits in a block is 26. The number of check bits in the block would be

(A) 3 **(B)** 4
(C) 5 **(D)** 7

h. Maximum-Length codes are generated by polynomials of the form

(A) $g(D) = h(D)\cdot\left(1+D^n\right)$ **(B)** $g(D) = \frac{h(D)}{\left(1+D^n\right)}$

(C) $g(D) = \frac{1+D^n}{h(D)}$ **(D)** None of these

i. If the data unit is 111111 and divisor is 1010, then the dividend at the transmitter is

(A) 1111111000 **(B)** 1111110000
(C) 111111 **(D)** 111111000

j. A source generates 4 messages. The entropy of source will be maximum when

(A) All probabilities are equal
(B) One of probabilities equal to 1 and others are zero
(C) The probabilities are $1/2$, $1/4$ and $1/2$
(D) Two probabilities are $1/2$ and others are zero

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Define the following terms:
(i) Joint probability (ii) Conditional probability
(iii) Probability mass function (iv) Statistical independence **(8)**

b. The input of a binary communication systems denoted by random variable X, takes on one of the two values 0 or 1 with probabilities $3/4$ and $1/4$ respectively. Due to error caused by noise in the system, the output Y differs from the input X occasionally. The behaviour of the communication system is modelled by the conditional probabilities.

$P(Y=1|X=1)=3/4$ & $P(Y=0|X=0)=\frac{7}{8}$

Find

(i) P(Y = 1) and P(Y=0)

(ii) $P(X=1|Y=1)$ **(8)**

Q.3 a. Explain the three models for continuous random variables. **(8)**

b. X and Y are two independent random variables, each having a Gaussian probability distribution function with a mean of zero and a variance of one.

(i) Find $P(|X|>3)$ using Q(4) and also obtain an upper bound. Given that $Q(0)=1/2, Q(3)=0.0013$.

(ii) Find the joint PDF of $Z=\sqrt{x^2+y^2}$ & $\omega=\tan^{-1}\left(y/x\right)$

(iii) Find P(z > 3) **(8)**

Q.4 a. Explain Markoff statistical model for information sources. **(8)**

b. A discrete source emits one of five symbols once every millisecond. The symbol probabilities are $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}$ and $\frac{1}{16}$ respectively. Find the source entropy and information rate. **(8)**

Q.5 a. Explain briefly Huffman coding and prefix coding. **(8)**

b. A source produces one of four possible symbols during each interval having probabilities $P(x_1)=\frac{1}{2}$, $P(x_2)=\frac{1}{4}$, $P(x_3)=P(x_4)=\frac{1}{8}$. Obtain the information contents of each of these symbols. **(8)**

Q.6 a. Explain Discrete Memoryless channel. **(8)**

b. A discrete memoryless source X has four symbols x_1, x_2, x_3, x_4 with probabilities $P(x_1)=0.4$, $P(x_2)=0.3$, $P(x_3)=0.2$, $P(x_4)=0.1$

(i) Calculate H(X)

(ii) Find the amount of information contained in the message $x_1x_2x_1x_3$ and $x_4x_3x_3x_2$. **(8)**

Q.7 a. Explain the following terms:

(i) Mutual information

(ii) Channel capacity **(8)**

b. Explain differential entropy and mutual information for continuous ensembles. **(8)**

Q.8 a. What is linear block code? Explain the steps for determination of all code words for a linear block code. **(8)**

b. The generator matrix for a (6, 3) block code is given as

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Find all code vectors of this code. **(8)**

Q.9 a. Explain cyclic codes. Give their advantages and disadvantages. **(8)**

b. Obtain the convolutional coded output for the message '1100101'. The convolutional encoder is shown in Fig.1 **(8)**

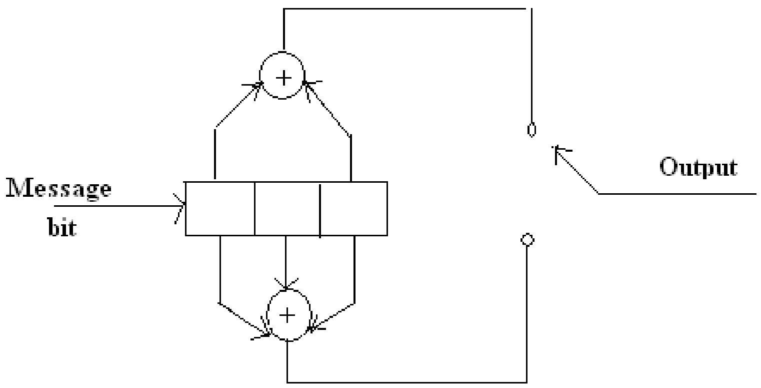

Fig.1