ROLL NO. _____

Code: AE57/AC57/AT57 Subject: SIGNALS AND SYSTEMS

AMIETE – ET/CS/IT

Time: 3 Hours **JUNE 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Given a unit step function u(t). Its time derivative is
(A) a unit impulse **(B)** another step function
(C) a unit ramp function **(D)** a sine function

b. The area under the curve $\int_{-\infty}^{\infty} \delta(t)dt$ is

(A) Infinity **(B)** Unity
(C) Zero **(D)** Undefined

c. A Periodic signal which can be expanded in Fourier series is
(A) A Power Signal **(B)** An Energy Signal
(C) Neither Energy nor Power signal **(D)** Real Signal

d. The DTFS coefficients of a real and even periodic signal are
(A) real and odd **(B)** imaginary and even
(C) real and even **(D)** imaginary and odd

e. The Fourier transform of sgn(t) is
(A) $2/j\omega$ **(B)** $1/j\omega$
(C) $4/j\omega$ **(D)** $j\omega$

f. The property of Fourier transform that states that the expansion in time domain is equivalent to compression in the frequency domain is

(A) Duality **(B)** Frequency Shifting
(C) Time Shifting **(D)** Time scaling

g. Flat Top Sampling leads to
(A) Aperture Effect **(B)** Aliasing
(C) Loss of Signal **(D)** Loss of higher frequency components

h. Laplace Transform converts convolution of time signals to
(A) Addition **(B)** Subtraction
(C) Multiplication **(D)** Division

i. The z-transform of $\delta(n-m)$ is
(A) z^{-n} **(B)** z^{-m}
(C) $1/(z-n)$ **(D)** $1/(z-m)$

j. The condition $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ must be satisfied by a system that is
(A) Causal **(B)** Linear
(C) BIBO Stable **(D)** Invertible

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Determine whether the following signals are periodic or not of periodic then find its fundamental period

(i) $x(n) = (-1)^{n^2}$

(ii) $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-2k)$ **(8)**

b. For each of the following systems determine whether it is Memoryless, Causal, Stable, Linear and Time invariant. **(8)**

(i) $y(n) = \log_e[x(n)]$

(ii) $y(n) = x(n^2)$

Q.3 a. Find the trigonometric Fourier series for the triangular wave shown in Fig.1 and hence plot its line spectrum. **(6)**

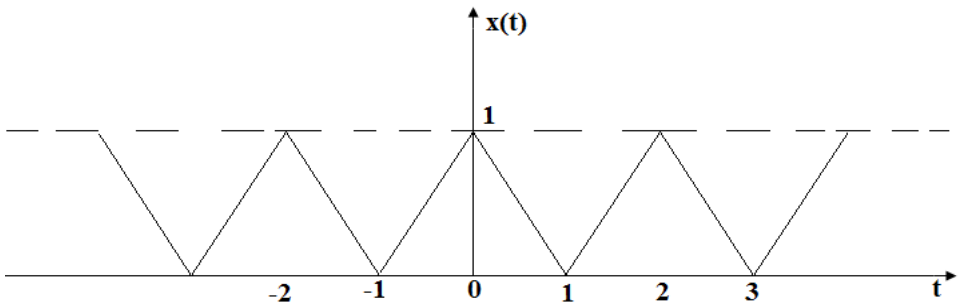

Fig.1

b. A continuous time periodic signal is real valued and has a fundamental period T= 8 .The non zero Fourier series coefficients for x(t) are $X_1 = X_{-1} = 2$,

$X_3 = X_{-3}^* = 4j$. Express x(t) in the form $x(t) = \sum_{n=0}^{\infty} A_n \cos(\omega_n t + \Phi_n)$ **(6)**

c. Find the time domain signal corresponding to following DTFS coefficients

$$X_k = \cos\left(\frac{k4\pi}{11}\right) + 2j\sin\left(\frac{k6\pi}{11}\right)$$ **(4)**

Q.4 a. State and Prove duality property of Continuous Time Fourier Transform. Using it, find the fourier Transform of following signals

(i) $g(t) = \frac{1}{1+jt}$

(ii) $x(t) = \frac{1}{1+t^2}$ **(8)**

b. Consider a stable LTI system characterized by the differential equation **(8)**

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

(i) Find the frequency response $H(\omega)$ and impulse response h(t) of the system.

(ii) What is the response of this system if the input $x(t) = e^{-t}u(t)$

Q.5 a. Suppose that a system has the response $(\frac{1}{4})^n u(n)$ to the input $(n+2)(\frac{1}{2})^n u(n)$ .If the output of this system is $\delta(n) - (-\frac{1}{2})^n u(n)$,what is the input? **(8)**

b. State and Prove convolution property of Discrete Time Fourier Transform. Using it determine the convolution $x(n) = x_1(n) * x_2(n)$ of the sequences, where

$$x_1(n) = x_2(n) = \delta(n+1) + \delta(n) + \delta(n-1)$$ **(8)**

Q.6 a. Determine the conditions on the sampling interval T_S so that each x(t) is uniquely represented by the discrete time sequence $x(n) = x(nT_S)$. **(8)**

(i) $x(t) = \cos(\pi t) + 3\sin(2\pi t) + \sin(4\pi t)$

(ii) $x(t) = \cos(2\pi t)\sin c(t) + 3\sin(6\pi t)\sin c(2t)$

b. A causal LTI system is described by the differential equation **(8)**

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Determine

(i) the frequency response of the system

(ii) the group delay associated with the system

(iii) output of the system to the input $x(t)=e^{-t}u(t)$

(iv) output of the system if the input has its fourier transform

$$X(jw) = \frac{j\omega + 1}{(j\omega + 2)}$$

Q.7 a. Consider the signal $x(t) = e^{-5t}u(t-1)$ and its Laplace Transform be X(s) **(8)**

(i) Evaluate X(s) and find its ROC

(ii) Determine the values of the finite numbers A and t_0 such that the Laplace transform G(s) of $g(t) = Ae^{-5t}u(-t-t_0)$ has the same algebraic form as X(s).What is the ROC corresponding to G(s)?

b. Find the inverse Laplace transform of $X(s) = \dfrac{-3}{(s+2)(s-1)}$

If the ROC is

(i) Re{s} > 1

(ii) Re{s} < - 2

(iii) -2 < Re{s} < 1 **(8)**

Q.8 a. Determine the signal x(n) whose z-transform is given by $X(z) = \log(1+az^{-1}), |z| > |a|$ **(8)**

b. Find the inverse z-transform of $X(z) = \dfrac{1+z^{-1}}{1-(1/3)z^{-1}}$

when

(i) ROC : |z| > 1/3

(ii) ROC : |z| < 1/3, using power series expansion **(8)**

Q.9 a. A random variable X has the uniform distribution given by

$$f_X(x) = \begin{cases} \dfrac{1}{2\pi}, & \text{for } 0 \le x \le 2\pi \\ 0, & \text{otherwise} \end{cases}$$

Determine its mean and variance **(8)**

b. A WSS random process X(t) with autocorrelation function $R_X(\tau) = e^{-a|\tau|}$ where a is a real positive constant is applied to the input of LTI system with impulse response $h(t) = e^{-bt}u(t)$ where b is real positive constant. Find the autocorrelation function of the output Y(t) of the system. **(8)**