ROLL NO. ___________________

Code: AC65 Subject: DISCRETE STRUCTURES

AMIETE – CS

Time: 3 Hours **JUNE 2013** Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If A×B = B×A, (where A and B are general matrices) then

(A) $A=\varphi$ **(B)** $A = B'$
(C) $B = A$ **(D)** $A' = B$

b. A partial order relation is transitive, reflexive and

(A) antisymmetric **(B)** bisymmetric
(C) antireflexive **(D)** asymmetric

c. $p \rightarrow q$ is logically equivalent to

(A) $\sim q \rightarrow p$ **(B)** $\sim p \rightarrow q$
(C) $\sim p \wedge q$ **(D)** $\sim p \vee q$

d. A recursive definition for the sequence $a_n = n(n+2)$ for $n \geq 1$ is

(A) $a_{n+1} = 2 \times a_n$ **(B)** $a_{n+1} = a_{n-1} + 2$
(C) $a_{n+1} = a_n + 2n + 3$ **(D)** $a_{n+1} = a_n + (2n+1)$

e. The number of distinct relations on a set of 3 elements is:

(A) 8 **(B)** 9
(C) 18 **(D)** 512

f. $(P \rightarrow q) \wedge (P \rightarrow R)$ is

(A) $P \rightarrow (Q \vee R)$ **(B)** $P \rightarrow (q \rightarrow R)$
(C) $P \rightarrow (q \wedge R)$ **(D)** $P \rightarrow (R \rightarrow q)$

g. Let N = {1, 2, 3…} be ordered by divisibility, which of the following subset is totally ordered

(A) (2, 6, 24) **(B)** (3, 5, 15)
(C) (2, 9, 16) **(D)** (4, 15, 30)

h. Three unbiased coins are tossed. What is the probability of getting at most two heads?

(A) 3/4 **(B)** 1/4
(C) 3/8 **(D)** 7/8

i. Context free languages are closed under

(A) union, intersection **(B)** intersection, complement
(C) union, kleene star **(D)** complement, kleene star

j. If any 6 numbers are chosen from 1 to 10, then two of them have their sum equal to

(A) 6 **(B)** 10
(C) 15 **(D)** 11

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Prove that $A - (B \cap C) = (A - B) \cup (A - C)$ **(5)**

b. A die is tossed thrice. Find the probability of getting an odd number at least once. **(5)**

c. A speaks truth in 80% of the cases and B speaks truth in 60% of the cases. Find the probability of the cases of which they are likely to contradict each other in stating the same fact. **(6)**

Q.3 a. Prove the following logical equivalences without using truth tables:
(i) $P \vee [P \wedge (P \vee Q)] \Leftrightarrow P$
(ii) $[P \vee Q \vee (\neg P \wedge \neg Q \wedge R)] \Leftrightarrow P \vee Q \vee R$
(iii) $[(\neg P \vee \neg Q) \to (P \wedge Q \wedge R)] \Leftrightarrow P \wedge Q$
(iv) $[(P \to q) \wedge (r \to q)] \Leftrightarrow [(P \vee r) \to q]$ **(8)**

b. What are tautologies and contradictions? Prove that, for any propositions P, Q, R the following compound propositions are tautologies:
(i) $[(P \to Q) \wedge (P \to R)] \to (P \to R)$
(ii) $[P \to (Q \to R)] \to [(P \to Q) \to (P \to R)]$ **(8)**

Q.4 a. Prove that $q \vee s$ is a valid conclusion from the premises $P \to q, r \to s, P \vee r$. **(8)**

b. Test the validity of the following arguement:
If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.
$\therefore$ I must have watched TV in the evenings. **(8)**

Q.5 a. Find an explicit definition of the sequence defined recursively by
$a_1 = 7, a_n = 2a_{n-1} + 1$ for $n \geq 2$ **(8)**

b. Prove by induction
1.1! + 2.2! + …….+n.n! = (n+1)! – 1 **(8)**

Q.6 a. If R is a relation defined by iff a + d = b + c, show that R is an equivalence relation. **(8)**

b. For any relation R in a set A, we can define the inverse relation R^{-1} by ${}^{a}R_{b}^{-1}$ iff ${}^{a}R_{b}$
Prove that
(i) As a subset of A x A, R^{-1}= {(b, a) /(a, b) $\in$ R}
(ii) R is symmetric iff R = R^{-1} **(8)**

Q.7 a. Prove that the inverse of a linear function is also linear and the two slopes are reciprocals of each other. **(8)**

b. A toy manufacturer has a new product to sell. The number of units to be sold, x, is a function of the price p: n(p)=30-25p . The revenue earned is a function of the number of units sold: r(x) = 1000-(1/4)x^2. Find the function for revenue in terms of price, p. **(8)**

Q.8 a. Let G be a cyclic group of order 6. How many of its elements generate G? **(8)**

b. State and prove Lagrange's theorem. Explain any two direct consequences of Langrange's theorem. **(8)**

Q.9 a. Let R be a ring with a unity element 1. Show that the set R* of units in R is a group under Multiplication. **(8)**

b. Give steps and an example to generate a parity check matrix. **(8)**