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## DipIETE - ET/CS (Current \& New Scheme)

Time: 3 Hours
JUNE 2015
Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q. 1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the $\mathbf{Q} .1$ will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.
Q. 1 Choose the correct or the best alternative in the following:
a. The value of $\frac{\lim }{\theta \rightarrow 0} \frac{\sin a \theta}{\sin b \theta}$, where a and b are fixed non zero real numbers, is
(A) 1
(B) $\frac{a}{b}$
(C) $\frac{-a}{b}$
(D) None of these
b. The value of $\frac{d}{d x}\left(\log _{a} x\right)$ is
(A) $\frac{1}{x}$
(B) $\frac{-1}{x}$
(C) $\frac{\log _{a} e}{x}$
(D) None of these
c. The value of $\int_{0}^{1} x e^{x} d x$ is
(A) 1
(B) 2
(C) -2
(C) None of these
d. If $A=$ diag. $(2,5,-3), B=$ diag. $(-1,6,4)$,then the value of $3 A-4 B+4 I$ is
(A) diag. (12, 7, -13)
(B) diag. $(-1,1,1)$
(C) diag. ( $0,1,0$ )
(D) None of these
e. The differential equation of the family of curves $y=e^{x}(A \cos x+B \sin x)$, where $A$ and $B$ are arbitrary constants, is
(A) $\frac{d y}{d x}+A x+B y=0$
(B) $\frac{d y}{d x}-A x-B y=0$
(C) $\frac{d^{2} y}{d x^{2}} 2 \frac{d y}{d x}+2 y=0$
(D) None of these
f. Three arithmetic means between 3 and 19 are
(A) 1, 2, 3
(B) 7, 11, 15
(C) 5, 7, 11
(D) None of these
g. If $\sin A=\frac{3}{5}$ and $\cos B=\frac{9}{41}$ where $0<A<\frac{\pi}{2}, 0<B<\frac{\pi}{2}$ then the value of $\cos (A-B)$ is
(A) $\frac{106}{107}$
(B) $-\frac{156}{205}$
(C) $\frac{156}{205}$
(D) None of these
$h$. The angle between the pair of line represented by $x^{2}-5 x y+4 y^{2}+3 x-4=0$ is
(A) $45^{\circ}$
(B) $\tan ^{-1}\left(\frac{3}{5}\right)$
(C) $\sin ^{-1}\left(\frac{1}{3}\right)$
(D) None of these
i. If $(-1,2)$ and $(4,-3)$ are the coordinates of the end points of the diameter of any circle, then the equation of circle is
(A) $x^{2}+y^{2}-3 x+y-10=0$
(B) $x^{2}+y^{2}=a^{2}$
(C) $x^{2}+y^{2}+2 g x+2 f y+c=0$
(D) None of these
j. The equation of tangent to the parabola $y^{2}=9 x$ at $(4,6)$ is
(A) $9 x-12 y+36=0$
(B) $x+y+a=0$
(C) $4 x+3 y-34=0$
(D) None of these


## Answer any FIVE Questions out of EIGHT Questions. <br> Each question carries 16 marks.

Q. 2 a. Differentiate the following functions:
(8)
(i) $x^{n} e^{x} \log _{e} x$
(ii) $\operatorname{cosec}^{-1}\left(\frac{x^{2}+1}{x^{2}-1}\right)+\cos ^{-1}\left(\frac{x^{2}-1}{x^{2}+1}\right)$
b. If $y=\sin \left(m \sin ^{-1} x\right)$, then prove that

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\begin{equation*}
\left(1-x^{2}\right) y_{n+1}=(2 n+1) x y_{n+1}+\left(n^{2}-m^{2}\right) y_{n} . \tag{8}
\end{equation*}
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Q. 3 Evaluate
a. $\int \frac{\sin x}{\sin (x-a)} d x$
b. $\int_{0}^{\frac{\pi}{2}} \cos ^{7} x d x$
Q. 4 a. Prove that the inverse of a square matrix, if it exists, is unique.
b. Find the adjoint of the matrice A, where
$A=\left[\begin{array}{lcr}1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1\end{array}\right]$
Q. 5 a. How many arrangements can be made with the letters of the word MATHEMATICS? In how many of them vowels are together?
b. Find the middle terms in the expansion of $\left(3 x-\frac{x^{3}}{6}\right)^{7}$.
Q. 6 a. Find the equation of the ellipse, whose foci and eccentricity are $( \pm 2,0)$ and $\frac{1}{2}$.
b. Find the equation to the circle, which passes through the point $(-2,4)$ and through the points in which the circle $x^{2}+y^{2}-2 x-6 y+6=0$ is cut by the line $3 x+2 y-5=0$.
Q. 7 a. Find the equation of the line which passes through the point $(3,4)$ and the sum of its intercepts on the axes is 14 .
b. Find the area of a triangle whose vertices are $\mathrm{A}(3,2), \mathrm{B}(11,8)$ and $\mathrm{C}(8,12) .(8)$
Q. 8 a. Find the differential equation of all circles in xy plane.
b. Solve the differential equation $\frac{d y}{d x}+\frac{1+\cos 2 y}{1-\cos 2 x}=0$.
Q. 9 a. Prove that $\cos 80^{\circ} \cos 60^{\circ} \cos 40^{\circ} \cos 20^{\circ}=\frac{1}{16}$
b. If $A+B+C=\pi$, Prove that $\sin 2 A+\sin 2 B+\sin 2 C=4 \sin A \sin B \sin C$

