

DipIETE – ET/CS {NEW SCHEME}

Time: 3 Hours

JUNE 2015

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of the $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$ is equal to

- (A) 0 (B) 1
(C) $-\frac{1}{3}$ (D) None of these

b. The value of $\int_0^{\frac{\pi}{2}} \cos^3 x \sin^4 x \, dx$ is equal to

- (A) $\frac{2}{35}$ (B) $\frac{3}{7}$
(C) $\frac{4}{7}$ (D) None of these

c. The solution of the equation $z^4 + 1 = 0$ is

- (A) $\frac{\pm 1 + i}{\sqrt{2}}$ (B) 1
(C) -1 (D) None of these

d. If the vector $2i - 4j + 5k$, $i - mj + k$ and $3i + 2j - 5k$ are coplanar, then the value of m will be

(A) $\frac{1}{2}$

(B) $\frac{26}{25}$

(C) $\frac{-1}{2}$

(D) None of these

e. The value of $\vec{A} \times (\vec{B} + \vec{C}) + \vec{B} \times (\vec{C} + \vec{A}) + \vec{C} \times (\vec{A} + \vec{B})$ is

(A) 0

(B) \vec{A}

(C) \vec{B}

(D) \vec{C}

f. The solution of $\frac{d^2 y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$ is

(A) 0

(B) $y = (c_1 + c_2)e^{4x}$

(C) -1

(D) None of these

g. The series $6 - 10 + 4 + 6 - 10 + 4 - 6 + 4 \dots \infty$ is

(A) Convergent

(B) Divergent

(C) Oscillatory Series

(D) None of these

h. A sequence which is monotonic and bounded is

(A) Convergent

(B) Divergent

(C) Oscillatory Series

(D) None of these

i. The Laplace Transform of $t^3 e^{-3t}$ is

(A) 1

(B) $\frac{s}{(s+3)^3}$

(C) $\frac{6}{(s+3)^4}$

(D) None of these

j. The Inverse Laplace Transform of $\frac{1}{s^n}$ exist only when the value of n is

(A) Positive Integer

(B) Negative Integer

(C) Zero

(D) None of these

Answer any FIVE Questions out of EIGHT Questions.

Each question carries 16 marks.

Q.2 a. By Using Taylor's series, calculate the value of $f\left(\frac{11}{10}\right)$, Where

$$f(x) = x^3 + 8x^2 + 15x - 24 \quad (8)$$

b. Evaluate $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{x} - \cot x \right)$ (8)

Q.3 a. Evaluate by using the reduction formula $\int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^4 \theta \cos 2\theta \, d\theta$ (8)

b. Find the common area lie between the parabolas $x^2 = ay$ and $y^2 = bx$ (8)

Q.4 a. State and prove De'Moviere's theorem. (8)

b. Separate the real and imaginary part of $\tan(x + iy)$ (8)

Q.5 a. Show that the vectors $\vec{A}, \vec{B}, \vec{C}$, if (8)

$$\vec{A} = 5\vec{i} + 6\vec{j} + 7\vec{k}, \vec{B} = 7\vec{i} - 8\vec{j} + 9\vec{k}, \vec{C} = 3\vec{i} + 20\vec{j} + 5\vec{k} \text{ are coplanar}$$

b. Prove that $\vec{i} \times (\vec{p} \times \vec{i}) + \vec{j} \times (\vec{p} \times \vec{j}) + \vec{k} \times (\vec{p} \times \vec{k}) = 2\vec{p}$ where $\vec{p} = p_1\vec{i} + p_2\vec{j} + p_3\vec{k}$ (8)

Q.6 a. Solve $\frac{d^2 y}{dx^2} + 6\frac{dy}{dx} + 9 = \frac{e^{-3x}}{x^3}$ (8)

b. An inductance of 2 henries and a resistance of 20 ohms are connected in series with e.m.f E Volts. If the current is zero when $t = 0$, find the current at the end of 0.01 sec, if $E = 100$ volts. (8)

Q.7 Examine the following series: (16)

$$(i) \sum \sqrt{(n^4 + 1)} - \sqrt{(n^4 - 1)}$$

$$(ii) \sum \frac{(n+1)^n}{n^{n+1}} x^2$$

Q.8 Find the Laplace Transform of $f(t)$, where (16)

$$(i) f(t) = \begin{cases} \frac{t}{a}, & \text{where } 0 < t < a \\ 1, & \text{where } a < t < \infty \end{cases}$$

$$(ii) f(t) = \frac{e^{-t} \sin t}{t}$$

Q.9 a. Find the Inverse Laplace Transform of $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$. (8)

b. Apply convolution theorem, find $L^{-1} \left\{ \frac{s^2}{(s^2 + a^2)^2} \right\}$ (8)