

Time: 3 Hours

**JUNE 2015**

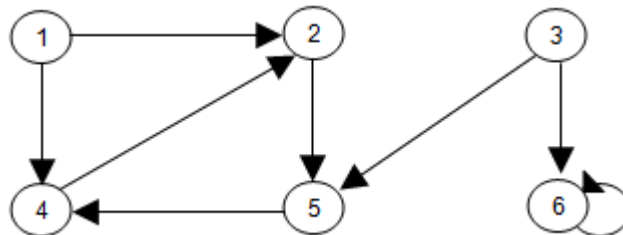
Max. Marks: 100

**PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.**

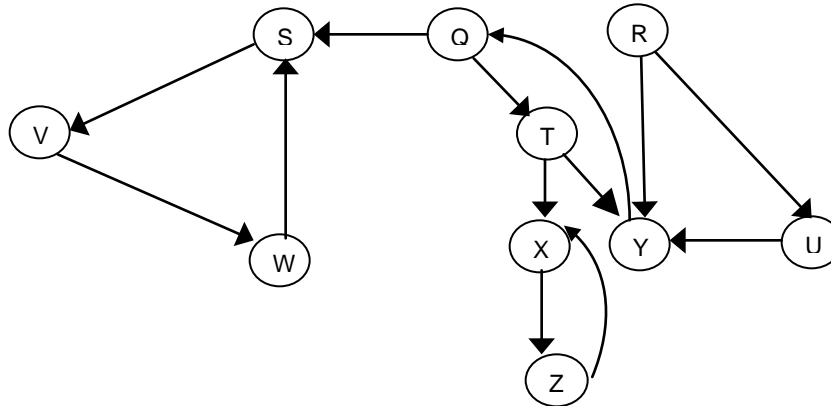
**NOTE:**

- Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.
- Parts of a question should be answered at the same place.

- Q.1**
- Find a solution to the 4-Queens problem using back-tracking techniques.
  - Give an Adjacency List representation for a complete binary tree on 15 vertices. Give an equivalent Adjacency Matrix representation. Assume that vertices are numbered from 1 to 15 as in a binary heap.
  - Illustrate the operation of Counting Sort on the array  $A=(6,0,2,0,13,4,6,1,3,2)$
  - Show all legal B-trees of minimum degree 2 that represent (1,2,3,4,5)
  - Give a simple example of a directed graph with negative weight edges for which Dijkstra's algorithm produces incorrect answers.
  - State Max Flow – Min Cut Theorem.
  - Show the comparisons the naive string matcher makes for the pattern  $P=0001$  in the text  $T = 000010001010001$  (7×4)
- Q.2**
- What are the different ways of representing a graph in the memory of a computer? Represent the following graph using any three ways. (9)



- b. Write an algorithm for Depth First Search on a directed graph. Show how your algorithm will work on the following graph. Your algorithm should consider the vertices in alphabetical order and assume that each adjacency list is ordered alphabetically. (9)



- Q.3** a. Write down the algorithm for Radix Sort. Illustrate your algorithm on the following list of English words: (9)

COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX

- b. Define Max-Heap and Min-Heap property. Write an algorithm for creating a max-Heap when the input is in the form of an array of integer numbers. Illustrate the operation of your algorithm on the array  $A = (27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0)$  (9)

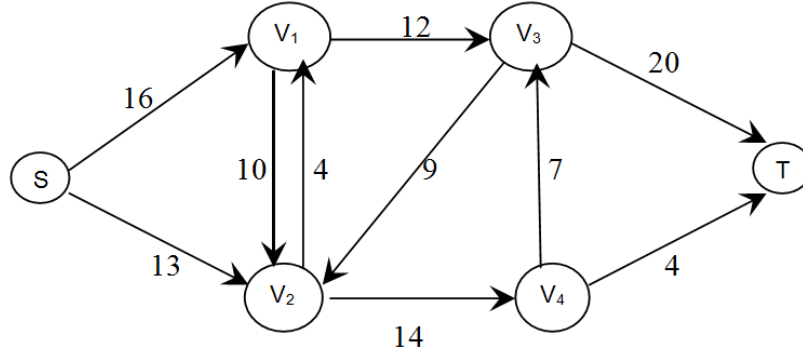
- Q.4** a. What do you mean by hashing? Explain any five popular hash functions. Draw the 11-item hash table resulting from hashing the keys 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5 using the hash function  $h(i) = (2i+5) \bmod 11$  (9)

- b. Taking a suitable example explain how disjoint sets are represented using linked lists. Show the data structure that results and the answers returned by the Find\_Set operations in the following programme. Use the linked-list representation with the weighted union heuristics. Assume that if the sets containing  $X_i$  and  $X_j$  have the same size, then Union ( $X_i, X_j$ ) appends  $X_j$ 's list on to  $X_i$ 's list. (9)

```

for i = 1 to 16
    Make -Set ( $X_i$ )
for i = 1 to 15 by 2
    Union ( $X_i, X_{i+1}$ )
for i = 1 to 13 by 4
    Union ( $X_i, X_{i+2}$ )
Union ( $X_1, X_5$ )
Union ( $X_{11}, X_{13}$ )
Union ( $X_1, X_{10}$ )
Find-Set ( $X_2$ )
Find-Set ( $X_9$ )
  
```

- Q.5** a. What do you understand by “Closest Pair of Points Problem”? How can you compute closet pair of points in (i)  $O(n^2)$  time (ii)  $O(n \log n)$  time. (6)
- b. Suppose that a graph  $G$  has a minimum spanning tree already computed. How quickly can we update the minimum spanning tree if we add a new vertex and incident edges of  $G$ . (6)
- c. Give an algorithm to compute the second best minimum spanning tree of a graph  $G$ . (6)
- Q.6** a. Write down the recursive formula that generates the optimal sub-structure of Longest Common Subsequence (LCS) problem. Determine an LCS of (b, a, a, b, a, b, a, b) and (a, b, a, b, b, a, b, b, a) (9)
- b. Write down Ford-Fulkerson method to find the maximum flow in a flow-network. Execute the algorithm on the following flow-network. (9)



- Q.7** a. Write down Knuith-Morris-Pratt(KMP) algorithm for string matching. Compute the prefix function for the pattern: ababbabbabbababbabb (12)
- b. Prove that (i) if any NP-complete problem is polynomial-time solvable, then  $P=NP$  (ii) if any problem in NP is not polynomial time solvable, then no NP-complete problem is polynomial time solvable. (6)