ROLL NO. ________________

Code: CT22 Subject: DISCRETE MATHEMATICAL STRUCTURES

ALCCS

Time: 3 Hours **JUNE 2015** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

Q.1

a. Verify the proposition $P \vee (P \wedge Q)$ is tautology.

b. “A Boolean algebra is a complemented, distributive lattice” Justify the statement.

c. Explain and compare Isomorphic graph and homorphic graph in detail with suitable examples

d. Explain Binary tree and draw two different binary trees with 5 nodes having maximum number of leaves.

e. Construct the tree of the following algebraic expression.
(i) $(A + (B - (C + B))) \div (3 \div (2 * A) * 5)$
(ii) $(5 - (3 - (9 * (7 - 2)))) * (2 - (3 + (9 + 5)))$

f. Construct an algorithm to determine if two given trees are identical.

g. Prove that, any connected graph with minimum numbers of edges will form a tree.

(7×4)

Q.2

a. Discuss the Demorgan’s law. How Tautologies is different from contradictions. Explain with the help of suitable example. **(6)**

b. Prove that $p \rightarrow (q \rightarrow r)$ and $(p \wedge \neg r) \rightarrow \neg q$ are logically equivalent. **(6)**

c. Let (S, R) be a poset. Show that (S, R^{-1}) is also a poset. (S, R^{-1}) is called as dual poset of (S, R). **(6)**

Q.3

a. Find DNF of the following identity without using truth table: **(8)**
$P \wedge (P \rightarrow Q)$
$(P \rightarrow Q) \wedge (\neg P \wedge Q)$
$(P \wedge \neg(Q \wedge R)) \vee (P \rightarrow Q)$

b. Convert the following SOP expression to POS
$ABC + AB'C' + AB'C + ABC' + A'B'C$ **(6)**

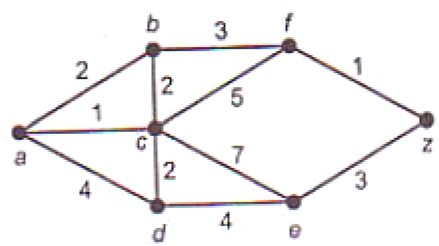
c. Define Boolean algebra and prove that the power set of any given set forms a Boolean algebra. **(4)**

Q.4 a. Let A = {1, 2, 3, 4, 6, 9} and relation R defined on A be "a divides b". Draw Hasse diagram for this relation. **(6)**

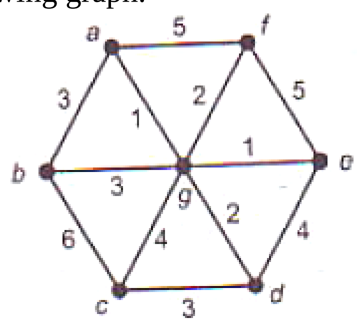
b. Explain the term equivalence class and partition and let say R be an equivalence relation on set A = {6, 7, 8, 9, 10} defined by R = {(6,6), (7,7), (8,8), (9,9), (10,10), (6,7), (7,6), (8,9), (9,8), (9,10), (10,9), (8,10), (10,8)}. Find the equivalence classes of R and hence find the partition of A corresponding to R **(8)**

c. Explain the term chromatic number and prove that chromatic number of complete bipartite graph $k_{m,n}$ where m and n are positive integers is two **(4)**

Q.5 a**.** Explain Dijkstra's algorithm and find the shortest path between the vertex *a* and vertex *z* using Dijkstra's algorithm for the following graph. **(4)**



b. Explain Kruskal's algorithm and find the minimum spanning tree using Kruskal's algorithm for the following graph. **(8)**



c**.** Draw the state transition diagram for the finite set of states, S = {s_0, s_1, s_2, s_3), a finite set of input alphabets, I = {a, b, c} and the transition function is given in table, **(6)**

	Input		
State	**a**	**b**	**c**
s_0	s_0	s_0	s_0
s_1	s_2	s_3	s_2
s_2	s_1	s_0	s_3
s_3	s_3	s_2	s_3

Q.6 a. Draw the diagraph of the machine whose state transition table is shown. Remember to label the edges with the appropriate inputs. Finite set machine, M = (S, I, F, s_0, F) where finite set of states, S ={s_0, s_1, s_2, s_3}, finite set of input alphabets, I ={0, 1} and transition function is given in the table, **(8)**

	Input	
State	**0**	**1**
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_0	s_0
s_3	s_2	s_1

b. Write short notes on the following: **(8)**
(i) Pigeon hole principle
(ii) Planer graph
(iii) Equivalence relation
(iv) Complete bipartite graph

c. Find the number of ways in which we can put 6 letters in 10 envelops. **(2)**

Q.7 a. Find a nonempty set and a relation on the set that satisfy each of the following combination of properties. Simultaneously draw a diagraph of each relation.
(i) Reflexive and symmetric but not transitive **(4)**
(ii) Reflexive and but not anti-symmetric **(4)**

b. Let *f* and *g* be the functions from the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Determine the compositions of *f* and *g* and of *g* and *f*. **(8)**

c. How many elements are there in the power set of set $A = \{\phi, \{\phi\}\}$? **(2)**