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## AMIETE - ET/CS/IT (Current \& New Scheme)

Time: 3 Hours
JUNE 2015
Max. Marks: 100
PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q. 1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the $\mathbf{Q} .1$ will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.
Q. 1 Choose the correct or the best alternative in the following:
a. The invariant points of the transformation $\mathrm{w}=\frac{5 \mathrm{z}+4}{\mathrm{z}+5}$ are
(A) $(2,2)$
(B) $(-2,-2)$
(C) $(2,-2)$
(D) None of these
b. The value of the integral $\int_{C} \frac{1+2 z}{z(1+z)} d z$, where $C$ is $|z|=\frac{1}{2}$, is
(A) $\pi \mathrm{i}$
(B) $2 \pi \mathrm{i}$
(C) $3 \pi i$
(D) 0
c. The residue of $\frac{\mathrm{ze}^{\mathrm{z}}}{(\mathrm{z}-1)^{3}}$ at its pole is
(A) 0
(B) $\frac{\mathrm{e}}{2}$
(C) e
(D) $\frac{3 \mathrm{e}}{2}$
d. If $\vec{A}$ is a constant vector and $\vec{R}=x i+y j+z k$, then $\operatorname{grad}(\vec{A} \cdot \vec{R})$ is equal to
(A) $\vec{A}$
(B) $\vec{R}$
(C) $\vec{A}+\vec{R}$
(D) $\vec{A} \cdot \vec{R}$
e. If $R=x i+y j+z k$, then $\int_{s} \vec{R} . d \vec{s}$, being the surface of unit sphere is
(A) $\pi$
(B) $2 \pi$
(C) $4 \pi$
(D) $8 \pi$


## Code: AE56/AC56/AT56

f. The missing term in the following table is:

| $\mathrm{x}:$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :---: |
| $\mathrm{f}(\mathrm{x}):$ | 1 | 3 | 9 | - | 81 |

(A) 33
(B) 31
(C) 30
(D) 27
g. The solution of $p \sqrt{x}+q \sqrt{y}=\sqrt{z}$ is
(A) $\sqrt{\mathrm{x}} \sqrt{\mathrm{y}}=\mathrm{f}(\sqrt{\mathrm{y}} \sqrt{\mathrm{z}})$
(B) $\sqrt{z}+\sqrt{\mathrm{y}}=\mathrm{f}(\sqrt{\mathrm{y}}+\sqrt{\mathrm{x}})$
(C) $\sqrt{\mathrm{z}}+\sqrt{\mathrm{x}}=\mathrm{f}(\sqrt{\mathrm{x}}+\sqrt{\mathrm{y}})$
(D) $\sqrt{z}-\sqrt{\mathrm{x}}=\mathrm{f}(\sqrt{\mathrm{x}}-\sqrt{\mathrm{y}})$
h. A speaks the truth in $75 \%$ cases and B in $80 \%$ of the cases. The percentage of cases they are likely to contradict each other in stating the same fact is
(A) $20 \%$
(B) $25 \%$
(C) $35 \%$
(D) $40 \%$
i. If $f(x)=\left\{\begin{array}{ll}\mathrm{kxe}^{-\mathrm{x}}, & \mathrm{x}>0 \\ 0, & \text { elsewhere }\end{array}\right.$, is the p.d.f. of x , then k is
(A) 1
(B) 2
(C) 4
(D) $\frac{1}{2}$
j. If a random variable has a Poisson distribution such that $P(1)=P(2)$, then mean of the distribution is
(A) 1
(B) 2
(C) 3
(D) 4

## Answer any FIVE Questions out of EIGHT Questions. <br> Each question carries 16 marks.

Q. 2 a. Show that every analytic function $w=f(z)$ define two families of curves, which from an orthogonal system.
b. Show that the transformation $\omega=\sin \mathrm{z}$, maps the families of lines $\mathrm{x}=$ constant and $y=$ constant into two families of confocal central conics.
Q. 3 a. If $f(\alpha)=\oint_{C} \frac{3 z^{2}+7 z+1}{z-\alpha} d z$, where $C$ is the circle $x^{2}+y^{2}=4$, find the values of (i) $\mathrm{F}^{\prime}(1-\mathrm{i})$ (ii) $\mathrm{F}^{\prime \prime}(1-\mathrm{i})$
b. Find the Laurent's series expression of $\frac{z^{2}-6 z-1}{(z-1)(z-3)(z+2)}$ in the region $3<|z+2|<5$.
Q. 4 a. Show that Curl Curl $\overrightarrow{\mathrm{F}}=\operatorname{grad} \operatorname{div} \overrightarrow{\mathrm{F}}-\nabla^{2} \overrightarrow{\mathrm{~F}}$.
b. Find the values of $\lambda$ and $\mu$ so that the surface $\lambda x^{2} y+\mu z^{3}=4$, may cut the surface $5 x^{2}=2 y z+9 x$ orthogonally at ( $1,-1,2$ ).
Q. 5 a. If $\vec{F}=\left(5 x y-6 x^{2}\right) i+(2 y-4 x) j$, evaluate $\int_{C} \vec{F} . d \vec{R}$ along the curve $C$ in the $x y-$ plane, $y=x^{3}$ from the point $(1,1)$ to $(2,8)$.
b. Use Divergence theorem to evaluate $\iint_{S} \vec{F} \cdot \overrightarrow{\mathrm{dS}}$, where $\overrightarrow{\mathrm{F}}=\mathrm{x}^{3} \mathrm{i}+\mathrm{y}^{3} j+\mathrm{z}^{3} k$, and $S$ is the surface of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$
Q. 6 a. Estimate the values of $f(22)$ and $f(42)$ from the following available data:

| $\mathrm{x}:$ | 20 | 25 | 30 | 35 | 40 | 45 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(\mathrm{x}):$ | 354 | 332 | 291 | 260 | 231 | 204 |

b. Find an approximate value of $\log _{e} 5$ by calculating to 4 decimal places, by Simpson's $\frac{1}{3}$ rule, $\int_{0}^{5} \frac{\mathrm{dx}}{4 \mathrm{x}+5}$, by dividing the range into 10 equal parts.
Q. 7 a. Solve the equation $\frac{y-z}{y z} p+\frac{z-x}{z x} q=\frac{x-y}{x y}$
b. Use Charpit's method to solve:
$2 \mathrm{zx}-\mathrm{px}^{2}-2 \mathrm{qxy}+\mathrm{pq}=0$
Q. 8 a. A student takes his examination in four subjects A, B, C, D. His chances of passing in A are $\frac{4}{5}$, in $\mathrm{B} \frac{3}{4}$, in C $\frac{5}{6}$ and in D are $\frac{2}{3}$. To qualify, he must pass in A and at least two other subjects. What is the probability that he qualifies.
b. A ball is transferred from an urn containing two white and three black balls to another urn containing four white and five black balls. A white ball is then taken out from second urn. What is the probability that the transferred ball is white?
Q. 9 a. The diameter of an electric cable is assumed to be a continuous variable with p.d.f, $f(x)=k x(1-x), 0 \leq x \leq 1$. Find the value of $k$ and also find mean and variance of $x$.
b. A Car- hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5 , calculate the proportion of days
(i) on which there is no demand
(ii) on which demand is refused $\left(\mathrm{e}^{-1.5}=0.2231\right)$

