ROLL NO. ___________________

Code: AE51/AC51/AT51/ AE101/AC101/AT101 Subject: ENGINEERING MATHEMATICS - I

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **JUNE 2015** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 TO Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTION FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If x = r cosθ and y = r sinθ, then $\frac{\partial(x, y)}{\partial(r, \theta)}$ is equal to

(A) r **(B)** θ
(C) r + θ **(D)** r - θ

b. The value of the double integral $\int_0^1 \int_{x^2}^{2-x} (xy)\, dxdy$ is

(A) $\frac{3}{4}$ **(B)** $\frac{3}{8}$
(C) $\frac{3}{16}$ **(D)** 3

c. The matrix $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular if x is equal to

(A) (3, 1) **(B)** (1, 2)
(C) (0, 3) **(D)** (2, 3)

d. Taylor's series solution of $\frac{dy}{dx}$ = - xy, y(0) = 1, upto x^4 is

(A) $y(x) = 1+\frac{x^2}{2}+\frac{x^4}{8}+$..........
(B) $y(x) = 1-\frac{x^2}{2}-\frac{x^4}{8}-$..........
(C) $y(x) = 1+\frac{x^2}{2}-\frac{x^4}{8}+$..........
(D) $y(x) = 1-\frac{x^2}{2}+\frac{x^4}{8}-$..........

e. If $z = \log\left(x^2 + xy + y^2\right)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(A) 0
(B) 1
(C) 2
(D) z

f. The solution of $\frac{d^2y}{dx^2} + 3a\frac{dy}{dx} - 4a^2y = 0$ is

(A) $y=C_1e^{-ax}+C_2e^{-4ax}$
(B) $y=C_1e^{ax}+C_2e^{4ax}$
(C) $y=C_1e^{ax}+C_2e^{-4ax}$
(D) $y=C_1e^{-ax}+C_2e^{4ax}$

g. The value of the integral $\int_0^{\pi/2} \sin^5 x \cos^3 x dx$ is equal to

(A) $\frac{3}{8}$
(B) $\frac{1}{8}$
(C) $\frac{1}{18}$
(D) $\frac{1}{24}$

h. If $P_3(x) = \frac{1}{2}\left(ax^3 + bx\right)$, then

(A) $x J_0 (x)$
(B) $x J_2 (x)$
(C) $-x J_0 (x)$
(D) $- x J_2 (x)$

i. $J_{½}(x)$ is equal to

(A) a = -3, b = 5
(B) a = 5, b = -3
(C) a = -3, b = -5
(D) a = 5, b = 2

j. If the product of two eigen values of matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16, then the third eigen value is

(A) 1 **(B)** 2
(C) 3 **(D)** 6

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If u = f(x, y), where x = Φ(t) and y = φ (t), show that $\frac{du}{dt} = \frac{\partial u}{dx}.\frac{dx}{dt} + \frac{\partial u}{\partial y}.\frac{dy}{dt}$ **(8)**

b. Expand f(x, y) = sin(xy) in powers of (x–1) and $\left(y - \frac{\pi}{2}\right)$ up to the second degree terms. **(8)**

Q.3 a. Change the order of integration and then evaluate $\int_0^{\infty}\int_0^{x} xe^{-\frac{x^2}{y}}\,dydx$ **(8)**

b. Find the volume enclosed by the cylinders $x^2+y^2 = 2ax$ and $z^2 = 2ax$ **(8)**

Q.4 a. Show that the equations 3x + 3y + 2z = 1, x + 2y = 4, l0y + 3z = -2, 2x – 3y – z = 5 are consistent and solve them. **(8)**

b. Find the eigen values and eigen vectors of the matrix. **(8)**

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Q.5 a. Solve the differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + \sin 2x$ **(8)**

b. Solve the differential equation $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\{\log(1+x)\}$ **(8)**

Q.6 a. Use the method of false position to find the fourth root of 32 correct to three decimal places. **(8)**

b. Apply Euler's method to solve for y at x = 0.1 from $\frac{dy}{dx} = x + y + xy, y(0) = 1$ taking step size h = 0.025 **(8)**

Q.7 a. Obtain the series solution of $\frac{d^2y}{dx^2} + x^2 y = 0$ **(8)**

b. Show that $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1)$ **(8)**

Q.8 a. Prove that $\int_{-1}^{1} P_m(n) P_n(n) dx = \begin{cases} 0, & m \neq n \\ \frac{2}{2n+1}, & m = n \end{cases}$ **(4+4)**

b. Prove that

$$J_4(n) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(n) + \left(1 - \frac{24}{x^2}\right) J_0(n)$$ **(8)**

Q.9 **(For Current Scheme students i.e. AE51/AC51/AT51)**

a. Solve the differential equation $(1 + x + y)^2 \frac{dy}{dx} = 1$ **(8)**

b. Find the orthogonal trajectories of family of circles $x^2 + y^2 = 2ax$ **(8)**

Q.9 **(For New Scheme students i.e. AE101/AC101/AT101)**

a. Find a Fourier Series to represent x^2 in the interval $(-\ell, \ell)$ **(8)**

b. Find the Fourier Cosine transform of e^{-x^2}. **(8)**