ROLL NO. _______________

Code: AC65 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current Scheme)

Time: 3 Hours **JUNE 2015** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Which of the following statement is the negation of the statement, "2 is even and -3 is negative"?

(A) 2 is even and -3 is not negative
(B) 2 is odd and -3 is not negative
(C) 2 is even or -3 is not negative
(D) 2 is odd or -3 is not negative

b. Let N = {1, 2, 3, ... } be ordered by divisibility, which of the following subset is totally ordered

(A) (2, 6, 24) **(B)** (3, 5, 15)
(C) (2, 9, 16) **(D)** (4, 15, 30)

c. If R is a relation "Less Than" from A = {1, 2, 3, 4} to B = {1, 3, 5} then RoR^{-1} is

(A) {(3,3), (3,4),(3,5)} **(B)** {(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)}
(C) {(3,3), (3,5),(5,3)(5,5)} **(D)** {(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)}

d. Seven (distinct) car accidents occurred in a week. What is the probability that they all occurred on the same day?

(A) 1 / 7^6 **(B)** 1/2^7
(C) 1/7^5 **(D)** 1/7^7

e. If A×B = B×A, (where A and B are general matrices) then

(A) A = φ **(B)** A = B'
(C) B = A **(D)** A' = B

f. The length of Hamiltonian path in a connected graph of n vertices is

(A) n-1 **(B)** n
(C) n+1 **(D)** n/2

g. Which of the following proposition is a tautology?

(A) $(p \vee q) \rightarrow p$ **(B)** $p \vee (q \rightarrow p)$
(C) $p \vee (p \rightarrow q)$ **(D)** $p \rightarrow (p \rightarrow q)$

h. Find the number of relations from A = {cat, dog, rat} to B = {male, female}

(A) 64 **(B)** 6
(C) 32 **(D)** 15

i. Let P(S) denotes the power set of set S. Which of the following is always true?

(A) P(P(S)) = P(S) **(B)** P(S) intersection S = P(S)
(C) P(S) intersection P(P(S)) = $\{\phi\}$ **(D)** $S \in P(S)$

j. If G is an undirected planner graph on n vertices with e edges then?

(A) e<= n **(B)** e<= 2n
(C) e<= 3n **(D)** none of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Check the validity of the following argument:- **(8)**
"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the labour market is not perfect."

b. Let L be a distributive lattice. Show that if there exists an a with **(8)**
$a \wedge x = a \wedge y$ and $a \vee x = a \vee y$, then x = y.

Q.3 a. Solve the recurrence relation **(8)**
T(k) = 2 T (k-1), T(0) = 1

b. What are the different types of quantifiers? Explain in brief. **(8)**
Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$

Q.4 a. If f is a homomorphism from a commutative semigroup (S,*) onto a semigroup (T, *'), then show that (T, *') is also commutative. **(8)**

b. How many words of 4 letters can be formed with the letters a, b, c, d, e, f, g and h, when **(8)**
(i) e and f are not to be included
(ii) e and f are to be included

Q.5 a. Let x be the set of all programs of a given programming language. Let R the relation on x defined as P1 R P2 if P1 and P2 give the same output on all the inputs for which they terminate. Is R an equivalence relation? If not which property fails? **(8)**

b. Prove that for every positive integer n, $n^3 - n$ is divisible by 3. **(8)**

Q.6 a. What are tautologies and contradiction? Prove that, for any propositions P, Q, R the following compound propositions are tautologies:

(i) $\left[(P \to Q) \wedge (P \to R)\right] \to (P \to R)$
(ii) $\left[P \to (Q \to R)\right] \to \left[(P \to Q) \to (P \to R)\right]$ **(8)**

b. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$. **(8)**

Q.7 a. Consider the function f : N $\to$N, where N is the set of natural numbers, defined by $f(n) = n^2 + n + 1$. Show that the function f is one-one but not onto. **(8)**

b. Using principles of inclusion and exclusion determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 or 7. **(8)**

Q.8 a. Find the orders of the groups $U(Z_{10})$, $U(Z_{11})$, and $U(Z_{12})$, and describe their structure. **(8)**

b. By finding a suitable generator, show that the multiplicative group of the field Z_{23} is cyclic. **(8)**

Q.9 a. Let G be the set of real numbers not equal to -1 and * be defined by a*b = a + b + ab. Prove that (G, *) is an abelian group. **(8)**

b. Prove that $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$ **(8)**