

Time: 3 Hours

**JUNE 2014**

Max. Marks: 100

**PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.**

**NOTE: There are 9 Questions in all.**

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a. The value of  $\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{x^2}$  is

(A)  $\frac{a^2 - b^2}{2}$

(B)  $\frac{b^2 - a^2}{2}$

(C)  $\frac{a^2 + b^2}{2}$

(D)  $a^2 + b^2$

b. If  $e^x(\sin x - \cos x)$  then  $\frac{dy}{dx}$  is equal to

(A)  $e^x \sin x \cdot \cos x$

(B)  $2e^x \sin x \cdot \cos x$

(C)  $2e^x \cos x$

(D)  $2e^x \sin x$

c. If  $z = 1 + i\sqrt{3}$ , then the value of  $z^2 + 4$  is equal to

(A)  $2z$

(B)  $3z$

(C)  $4z$

(D)  $z$

d. The expression  $6e^{5\pi i/6}$  is the form of  $(a + ib)$  is equal to

(A)  $3\sqrt{3} + i$

(B)  $3\sqrt{3} + 3i$

(C)  $-3\sqrt{3} + 3i$

(D)  $3\sqrt{3} - 3i$

e. If the position vector  $\vec{a}$  of the point  $(5, n)$  is such that  $|\vec{a}| = 13$ , then the value of  $n$  is equal to

(A)  $\pm 13$

(B)  $\pm 11$

(C)  $\pm 14$

(D)  $\pm 12$



**Q.4** a. If  $|z_1 + z_2| = |z_1 - z_2|$ , prove that the difference of amplitudes of  $z_1$  and  $z_2$  is  $\frac{\pi}{2}$ . (8)

b. A resistance of 20 ohms an inductance of 0.2 Henry and a capacitance of 100 micro-farad are connected in series across 220 volts, 50 cycle/sec main. Calculate, (i) Impedance, (ii) Current, (iii) Voltage across, L, R and C. (8)

**Q.5** a. A rigid body is rotating with angular velocity 2 radian / sec about an axis OR, where R is  $2i - j + k$  and O is the origin. Find the velocity of the point  $3i + 2j - k$  on the body. (8)

b. A force of 15 units acts parallel to the line  $i - 2j + 2k$  and passes through the points  $2i - 2j - k$ . Using vector method, find the magnitude of the moment of the force about the point  $i + j - k$ . (8)

**Q.6** a. Solve the differential equation  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x$  (8)

b. A body weighing 10 kg is hung from a spring. A pull of 20 kg. Wt. will stretch the spring to 10 cm. The body is pulled down to 20 cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position at time t sec., the maximum velocity and the period of oscillation. (8)

**Q.7** a. Find a Fourier series for the function  $f(x) = x$  in the interval  $[-\pi, \pi]$  (8)

b. Develop  $f(x)$  as Fourier series in the interval (0, 2), if  $f(x) = \begin{cases} \pi x, 0 \leq x \leq 1 \\ \pi(2-x), 1 \leq x \leq 2 \end{cases}$  (8)

**Q.8** a. Find the Laplace transform of  $\cos t \cdot \cos 2t \cdot \cos 3t$ . (8)

b. Find Laplace transform of  $\cos^4 t$  (8)

**Q.9** a. Find  $L^{-1} \left\{ \frac{3s - 2}{s^2 - 4s + 20} \right\}$  (8)

b. Solve the equation  $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, x(0) = 0, x'(0) = 1$   
Using Laplace transform. (8)