

**DIPIETE – ET/CS {NEW SCHEME}**

Time: 3 Hours

**JUNE 2014**

Max. Marks: 100

**PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.**

**NOTE: There are 9 Questions in all.**

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a.  $\text{Lt}_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$  is equal to

- (A) 3 (B) -2  
(C) 2 (D) -3

b. If  $y = (x + 1)(x + 2)$ , then  $\frac{dy}{dx}$  is

- (A)  $3x + 2$  (B)  $2x + 3$   
(C)  $2x - 3$  (D)  $3x - 2$

c.  $\int \sin^2 x \cdot \cos^2 x \, dx$  is equal to

- (A)  $\frac{1}{8} \left( x + \frac{\cos 4x}{4} \right) + C$  (B)  $\frac{1}{8} \left( x - \frac{\cos 4x}{4} \right) + C$   
(C)  $\frac{1}{8} \left( x + \frac{\sin 4x}{4} \right) + C$  (D)  $\frac{1}{8} \left( x - \frac{\sin 4x}{4} \right) + C$

d. If  $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & zw \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$  then x, y, z and w is equal to

- (A) 2, 4, 1, 3 (B) 1, 2, 3, 4  
(C) 3, 2, 4, 1 (D) 4, 3, 2, 1

e. If  $\Delta = \begin{bmatrix} x-2 & -3 \\ 3x & 2x \end{bmatrix} = 3$ , then the value of x is

(A)  $\frac{1}{2}, 3$

(B)  $\frac{1}{2}, -3$

(C)  $-\frac{1}{2}, 3$

(D)  $-\frac{1}{2}, -3$

f.  $A = \cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$  and  $B = \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$  then  $A+B$  is equal to

(A)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(B)  $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(C)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(D)  $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

g. The value of  $2 \cos\left(\frac{\pi}{13}\right) \cdot \cos\left(\frac{9\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right)$  is equal to

(A)  $\infty$

(B) 1

(C) 0

(D) 2

h. If 17<sup>th</sup> and 18<sup>th</sup> terms in the expansion of  $(2+a)^{50}$  are equal then the value of  $a$  is

(A) 1

(B) -1

(C) 2

(D) -2

i. If  $\tan \alpha = \frac{a}{a+1}$ ,  $\tan \beta = \frac{1}{2a+1}$ , then  $(A+B)$  is equal to

(A)  $\tan^{-1}\left(\frac{\pi}{4}\right)$

(B)  $\tan^{-1}\left(\frac{\pi}{2}\right)$

(C)  $\tan^{-1}\left(\frac{\pi}{3}\right)$

(D)  $\tan^{-1} \pi$

j. The slop of the line joining between the pts. (1,2) and (4,5) is equal to

(A) 2

(B) -1

(C) 1

(D) -2

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**Answer any FIVE Questions out of EIGHT Questions.**  
**Each question carries 16 marks.**

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**Q.2** a. Prove that  $\cos A \cdot \cos(60^\circ - A) \cdot \cos(60^\circ + A) = \frac{1}{4} \cos 3A$  and deduce that

$$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ = \frac{1}{8} \quad (8)$$

b. If A, B, C are the angle of a triangle, then prove that  $\cos A + \cos B + \cos C = 1 + 4 \sin A/2 \cdot \sin B/2 \cdot \sin C/2$  (8)

**Q.3** a. Find the co-efficient of  $x^{32}$  in the expansion of  $\left(x^4 - \frac{1}{x^3}\right)^{15}$ . Also find co-efficient of  $x^{-17}$ . (8)

b. The sum of first three terms of a G.P. is 16 and the sum of the next three term is 128. Find the sum of the  $n^{\text{th}}$  term of GP. (8)

**Q.4** a. Show that,  $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan \theta/2 \\ \tan \theta/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta/2 \\ -\tan \theta/2 & 1 \end{bmatrix}^{-1}$  (8)

b. Find the matrix A satisfying the equation  $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} A \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  (8)

**Q.5** a. For what values of K are the three lines  $4x + 7y - 9 = 0$ ,  $5x + ky + 15 = 0$  and  $9x - y + 6 = 0$  are concurrent. (8)

b. Find the equation of the two lines passing through the point (1, -1) and inclined at an angle of  $45^\circ$  with the line  $2x - 5y + 7 = 0$  (8)

**Q.6** a. Find the equation of the circle which passes through the intersection of two circles,  $x^2 + y^2 - 8x - 24y + 7 = 0$ , and  $x^2 + y^2 - 4x + 10y + 8 = 0$  and has its centre on the x-axis. (8)

b. Find the centre, length of the axes, eccentricity, directrices, foci and the length of the latus rectum of the hyperbola  $9x^2 - 16y^2 = 144$  (8)

**Q.7** a. If  $y = x \log \frac{x-1}{x+1}$ , show that  $y_n = (-1)^{n-2} (n-2)! \left[ \frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n} \right]$  (8)

b. Find the value of x for which the function  $(x-2)^3 \cdot (x-3)^2$  is a maximum or minimum. (8)

**Q.8** a. Evaluate  $\int \frac{2x}{(x^2+1)(x^2+2)} dx$  (8)

b. Evaluate  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$  (8)

**Q.9** a. Solve the initial value problem  $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ , when  $y(0) = 0$ . (8)

b. Solve  $x(y-x) \frac{dy}{dx} = y(y+x)$  (8)