Code: AE77/AC77 Subject: DIGITAL SIGNAL PROCESSING

AMIETE – ET/CS

Time: 3 Hours **JUNE 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The signal $x(n)=\sum_{n=-\infty}^{\infty}\delta(n)$ is

(A) ∞ **(B)** 0
(C) 1 **(D)** undefined

b. The quantization step size Δ for an analog signal of range R and binary word size b is

(A) $\frac{R}{2}$ **(B)** R^b
(C) $\frac{R}{2^b}$ **(D)** $2^b R$

c. A discrete-time periodic signal x(n) having a period N is convolved with itself. The resulting signal is

(A) not periodic **(B)** periodic having a period N
(C) periodic having a period $2N$ **(D)** periodic having a period $N/2$

d. If the Fourier series coefficients of a signal are periodic then the signal must be

(A) continuous-time, periodic **(B)** continuous-time, non-periodic
(C) discrete-time, non-periodic **(D)** discrete-time, periodic

e. The Fourier transform of a real and odd signal x(n) is

(A) imaginary and odd function of frequency
(B) complex, in general
(C) real and odd function of frequency
(D) purely imaginary

f. In an N -point DFT of finite duration sequence $x(n)$ of length L, the value of N should be

(A) $N \geq L$ **(B)** $N < L$

(C) $N = 0$ **(D)** $N = L^2$

g. In a 32-point DFT by radix-2 FFT, there are __________ stages of computations with __________ butterflies per stage.

(A) thirty two, five **(B)** sixteen, five

(C) five, thirty two **(D)** five, sixteen

h. The algorithm used to compute any set of equally spaced samples of Fourier transform on the unit circle is

(A) DFT algorithm **(B)** FFT algorithm

(C) Goertzel algorithm **(D)** chirp-z transform algorithm

i. The region of convergence of the signal $x(n) = \{1, 2, \underline{8}, 4, 6\}$ is

(A) all z except $z = 0$ and $z = \infty$ **(B)** all z except z = 0

(C) all z except $z = \infty$ **(D)** all z

j. A system characterized by the system function $H(z) = \frac{1}{2}(1 - z^{-1})$ is a

(A) lowpass filter **(B)** highpass filter

(C) bandpass filter **(D)** bandreject filter

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. The signals $x_1(t) = 10\cos(100\pi t)$ and $x_2(t) = 10\cos(50\pi t)$ are both sampled at $f_s = 75\,\text{Hz}$. Show that the two sequences of samples so obtained are identical. **(8)**

b. Define quantization and quantization error? Derive signal to quantization noise ration for sinusoidal signals. **(8)**

Q.3 a. A discrete-time causal LTI system has the system function **(8)**

$$H(z) = \frac{(1 + 0.2z^{-1})(1 - 9z^{-2})}{1 + 0.81z^{-2}}$$

(i) Is the system stable?

(ii) Find expressions for a minimum-phase system $H_{min}(z)$ and an all pass system $H_{ap}(z)$ such that $H(z) = H_{min}(z)H_{ap}(z)$

b. A nonminimum-phase causal signal x(n) has z-transform **(8)**

$$X(z)=\frac{\left(1-\frac{3}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)\left(1+\frac{5}{3}z^{-1}\right)}{(1-z^{-1})^2\left(1-\frac{1}{4}z^{-1}\right)}$$

For what values of the constant β will the signal $y(n)=\beta^n x(n)$ be minimum-phase?

Q.4 a. Find the 10-point inverse DFT of $X(k)=1+2\delta(k)$. **(6)**

b. Consider the length-12 sequence, defined for $0\le n\le 11$, **(10)**
$x(n)=\{\underline{3},-1,2,4,-3,-2,0,1-4,6,2,5\}$ with a 12-point DFT given by $X(k)$, $0\le k\le 11$. Evaluate the following functions of $X(k)$ without computing the DFT:

(i) $X(0)$ (ii) $X(6)$ (iii) $\sum_{k=0}^{11} X(k)$ (iv) $\sum_{k=0}^{11} e^{-j4\pi k/6}X(k)$ (v) $\sum_{k=0}^{11} |X(k)|^2$

Q.5 a. What is FFT? Develop DIT-FFT algorithm for $N=8$ and draw signal flow graph. **(8)**

b. Let $x(n)$ be a real-valued N-point sequence $\left(N=2^m\right)$. Develop a method to compute an N-point DFT $X'(k)$, which contains only the odd harmonics [i.e., $X'(k)=0$ if k is even] by using only a real $\frac{N}{2}$-point DFT. **(8)**

Q.6 a. Discuss the factors that influence the choice of structure for realization of a LTI system. **(6)**

b. Obtain two canonical realizations of the system function:

$$H(z)=\frac{1+2z^{-1}-z^{-2}}{1+z^{-1}-z^{-2}}$$ **(10)**

Q.7 a. Using a rectangular window, design a lowpass filter with a passband gain of unity, cutoff frequency of 1000 Hz and working at a sampling frequency of 5 KHz. Take the length of the impulse response as 7. **(8)**

b. Explain the mapping of s-plane to z-plane using bilinear transformation with respect to IIR filter design. **(8)**

Q.8 a. Consider a sequence $x(n)=\{\underline{1},2,3,4\}$ its DFT is given by $x(k)=\{\underline{10},-2+j2,-2,-2-j2\}$. The sampling rate is 10 Hz. **(8)**

(i) Determine the sampling period, time index and sampling time instant for a discrete time sample x(3) in time domain.

(ii) Determine the frequency resolution, frequency bin number and frequency for each of the DFT coefficients X(1) and X(3) in frequency domain.

b. Write technical note on time-dependent Fourier transform. **(8)**

Q.9 a. Write technical note on digital Hilbert transformer and its applications. **(8)**

b. Consider a sequence $x(n)$ with DTFT $X(e^{j\omega})$. The sequence $x(n)$ is real valued and causal and $X_R(e^{j\omega}) = 2 - 2a\cos(\omega)$. Determine $X_I(e^{j\omega})$. **(8)**