ROLL NO. ________________

**Code: AE73** **Subject: INFORMATION THEORY & CODING**

# AMIETE – ET

**Time: 3 Hours** **JUNE 2014** **Max. Marks: 100**

***PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.***

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

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**Q.1 Choose the correct or the best alternative in the following:** **($2\times10$)**

a. The law of probability says that $P(A\cup B) =$

**(A)** P(A) + P(B) – P(AB) **(B)** P(A) $\geq$ 1 & P(B) $\geq$ 1
**(C)** 1 – P(AB) **(D)** P (A) + P(B) + P(AB)

b. The central limit theorem implies that a random variable which is determined by a large number of independent causes tends to have a

**(A)** NORMAL DISTRIBUTION
**(B)** GAUSSIAN DISTRIBUTION
**(C)** WHITE NOISE DISTRIBUTION
**(D)** UNIFORM DISTRIBUTION

c. A discrete source emits one of five symbols once every millisecond. The symbols probabilities are $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}$ & $\frac{1}{16}$ respectively. The source entropy is

**(A)** 1.875 bits/symbol **(B)** 1875 bits/sec
**(C)** 18.75 bits/symbol **(D)** 18.75 bits/sec

d. Prefix codes are also referred to as
**(A)** Binary codes **(B)** Inequality codes
**(C)** Huffman codes **(D)** Instantaneous codes

e. The mutual information of a channel is always
**(A)** Asymmetric & Negative **(B)** Symmetric & Non - Negative
**(C)** Uncertain **(D)** Unequal

f. According to channel capacity theorem, the channel capacity per unit time is expressed as:

**(A)** $C = \log_2\left(1+\frac{P}{N_0 B}\right)$ bits/sec. **(B)** $C = \frac{1}{2}\log_2\left(1+\frac{P}{N_0 B}\right)$ bits/sec.

**(C)** $C = B\log_2\left(1+\frac{P}{N_0 B}\right)$ bits/sec. **(D)** $C = B\log_2\left(1+\frac{P}{N_0}\right)$ bits/sec.

g. Error control codes are often divided into two broad categories:

**(A)** Random codes & Impulse codes **(B)** Huffman & Hamming codes
**(C)** Block codes & Continuous codes **(D)** Block codes & Convolutional codes

h. Consider a (15, 7) BCH code generated by $g(x) = x^8 + x^4 + x^2 + x + 1$. This code has a minimum distance of 5. Hence it is
**(A)** Single error correcting code **(B)** Double error correcting code
**(C)** Triple error correcting code **(D)** Quadruple error correcting code

i. A random variable X is said to have a uniform probability Distribution function if

**(A)** $f_X(X) = \begin{cases} \frac{1}{b+a}, & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$ **(B)** $f_X(X) = \begin{cases} \frac{b+a}{2}, & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$

**(C)** $f_X(X) = \begin{cases} \frac{b-a}{2}, & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$ **(D)** $f_X(X) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$

j. The square root of variance is called
**(A)** Standard Deviation **(B)** Mean
**(C)** Co-variance **(D)** Chebyshev's Inequality

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**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

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**Q.2** a. Explain the following terms: **(8)**
(i) Outcome
(ii) Random Event
(iii) Mutually Exclusive Event
(iv) Marginal Probability

b. Binary data are transmitted over a noisy communication channel in blocks of 16 binary digits. The probability that received binary digit is in error due to channel noise is 0.1. Assume that the occurrence of an error in a particular digit does not influence the probability of occurrence of an error in any other digit within the block.
(i) Find the average number of errors per block
(ii) Find the variance of the number of error per block
(iii) Find the probability that the number of errors per block is greater than or equal to 5. **(8)**

**Q.3** a. X and Y are two independent random variables, each having a Guassian pdf with a mean of zero and variance of one.
Find $P(|X| > 3)$ using the value of Q(y) given Q(3) = 0.0013 **(5)**

b. What is a Random Process? X is a Gaussian random variable with $\mu_X = 2$ and $\sigma_X^2 = 9$. Find $P(-4 < X \le 5)$. **(5)**

c. Explain: Stationarity, Time Averages & ergodicity **(6)**

**Q.4** a. Give an example of a Markoff source. Draw a tree diagram for the same. **(8)**

b. Consider an information source modelled by a discrete ergodic Markoff random process whose graph is shown below in Fig.1.

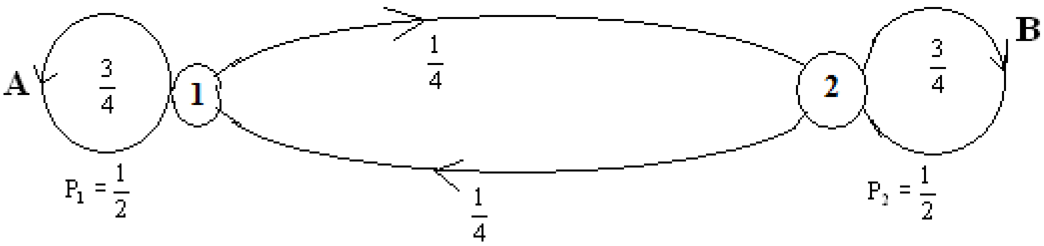


**Fig.1**

Find the source entropy H and average information content symbol in messages containing one and two symbols i.e. Find $G_1$ and $G_2$ **(8)**

**Q.5** a. What do you understand by source Encoding? State source coding theorem and find the relation between efficiency, entropy and average codeword length. **(8)**

b. Determine the Huffmen code for the following massage with their probabilities given.

| $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $m_7$ |
|---|---|---|---|---|---|---|
| 0.05 | 0.15 | 0.2 | 0.05 | 0.15 | 0.3 | 0.1 |

Find the average length $\overline{L}$ and the efficiency of the code. **(8)**

**Q.6** a. Explain the channel coding theorem. What is its application to Binary Symmetric channels? **(8)**

b. Calculate the capacity of a lowpass channel with a usable bandwidth of 3000 Hz and $S/N = 10^3$ at the channel output. Assume the channel noise to be Gaussian and white. **(8)**

**Q.7** a. The capacity of a channel with bandwidth B and additive Gaussian band limited white noise is

$$C = B \log_2\left(1 + \frac{S}{N}\right) \text{ bits/sec}$$

Where S and N are average signal power and noise power, respectively at output of channel.
What is this theorem referred as and what are its two important implications to communication systems? **(8)**

b. A CRT terminal is used to enter alphanumeric data into a computer. The CRT is connected to the computer through a voice grade telephone line having a usable bandwidth of 3000 Hz and an output SNR of 10 dB. Assume that terminal has 128 characters and data sent from terminal consist of independent sequences of equiprobable characters.
(i) Find the capacity of channel.
(ii) Find the maximum rate (theoretical) at which data can be transmitted from the terminal to computer without errors. **(8)**

**Q.8** a. A linear block code with a minimum distance $d_{min}$ can correct upto $\left[(d_{min}-1)/2\right]$ errors and detect upto $d_{min}$-1 errors in each codeword, where $\left[(d_{min}-1)/2\right]$ denotes the largest integer no greater than $(d_{min}-1)/2$. Give the proof of this theorem. **(7)**

b. Consider a (7, 4) Linear code whose generator matrix is

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

(i) Find all code vectors of this code
(ii) Find the parity check matrix for this code
(iii) Find $d_{min}$. **(9)**

**Q.9** a. The generator polynomial of a (7, 4) cyclic code is $g(x) = 1 + x + x^3$. Find all the code words. **(8)**

b. Explain maximum likelihood decoding of convolutional codes. **(8)**