ROLL NO. ________________

Code: AE56/AC56/AT56 Subject: ENGINEERING MATHEMATICS - II

# AMIETE – ET/CS/IT

**Time: 3 Hours** **JUNE 2014** **Max. Marks: 100**

***PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.***

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

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**Q.1 Choose the correct or the best alternative in the following:** **($2\times10$)**

a. Which one of the following functions is harmonic-

**(A)** u (x, y) = 4xy – 3x + 2 **(B)** u (x, y) = $x^2 + y^2$

**(C)** u (x, y) = $x^2y + xy^2$ **(D)** u (x, y) = $xy - x^2y^2$

b. The value of the integral $\int_C \frac{dz}{z-4}$, where c is $|z| = 1$, is

**(A)** $2\pi\ i$ **(B)** $4\ \pi\ i$

**(C)** $-4\ \pi\ i$ **(D)** 0

c. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$, where t is the time. The component of its velocity at t = 1 in the direction $\hat{i} + \hat{j} + 3\hat{k}$ is equal to

**(A)** 11 **(B)** $\sqrt{11}$

**(C)** $\frac{1}{\sqrt{11}}$ **(D)** None of these

d. The value of integral $\int_0^1 (e^t\hat{i} + e^{-2t}\hat{j} + t\,\hat{k})dt$ is equal to

**(A)** $e\hat{i} - (e^2 - 1)\hat{j} + \hat{k}$ **(B)** $(e-1)\hat{i} - (e^2 - 1)\hat{j} - \hat{k}$

**(C)** $(e-1)\hat{i} - \frac{1}{2}(e^{-2} - 1)\hat{j} + \frac{1}{2}\hat{k}$ **(D)** 0

e. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then the value of div $\vec{r}$ is equal to

**(A)** 0 **(B)** 1
**(C)** 2 **(D)** 3

f. The value of $\Delta^3(1\text{-}x)(1\text{-}2x)(1\text{-}3x)$ is

**(A)** -6 **(B)** 3
**(C)** 2 **(D)** -36

g. The partial differential equation by eliminating arbitrary function 'f' from $z = e^y f(x + y)$ is

**(A)** $\frac{\partial z}{\partial y} = y + z - \frac{\partial z}{\partial x}$ **(B)** $\frac{\partial z}{\partial y} = z + \frac{\partial z}{\partial x}$
**(C)** $\frac{\partial z}{\partial x} = x - \frac{\partial z}{\partial y}$ **(D)** $\frac{\partial z}{\partial x} \frac{\partial z}{\partial y} = x$

h. If a card is drawn from a deck of playing cards, then the probability that it will be either the king of Diamonds or the Queen of Hearts is equal to

**(A)** $\frac{1}{52}$ **(B)** $\frac{1}{2}$
**(C)** $\frac{1}{13}$ **(D)** $\frac{1}{26}$

i. The probability of throwing 9 with two dice is equal to

**(A)** $\frac{1}{9}$ **(B)** $\frac{1}{36}$
**(C)** $\frac{1}{4}$ **(D)** $\frac{1}{18}$

j. If the probability of a defective bolt is 0.1, then the mean for the distribution bolts in a total of 400 is

**(A)** 4 **(B)** 40
**(C)** 400 **(D)** 0.4

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**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

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**Q.2** a. Show that the real and imaginary parts of the function w = log z satisfy the Cauchy-Riemann equations when z is not zero. **(8)**

b. Find the bilinear transformation which maps z = 1, i, -1 onto w = i, 0, -i respectively. **(8)**

**Q.3** a. Evaluate the following integral using Cauchy integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z| = \frac{3}{2}$. **(8)**

b. Obtain the Taylor's or Laurent's series which represents the function

$f(z) = \frac{1}{(1+z^2)(z+2)}$

when-

(i) $1<|z|<2$ (ii) $|z|>2$ **(8)**

**Q.4** a. Find the directional derivative of the divergence o $f(x,y,z) = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$ at the point (2, 1, 2) in the direction of the outer normal to the sphere, $x^2 + y^2 + z^2 = 9$ **(8)**

b. A vector $\vec{r}$ is defined by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. If $|\vec{r}|$ = r then show that the vector $r^n\vec{r}$ is irrotational. **(8)**

**Q.5** a. Evaluate by Green's theorem in plane $\int_c (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy)$

Where C is the rectangle with vertices (0, 0), ($\pi$, 0), ($\pi, \frac{1}{2}\pi$), (0, $\frac{1}{2}\pi$) **(8)**

b. Using Stoke's theorem, evaluate $\int_c \left[(2x-y)dx - yz^2dy - y^2zdz\right]$ where C is the circle $x^2 + y^2 = 1$, corresponding to the surface of sphere of unit radius. **(8)**

**Q.6** a. Determine f(x) as a polynomial in for the following data: **(8)**

| | | | | | |
|---|---|---|---|---|---|
| x: | -4 | -1 | 0 | 2 | 5 |
| F(x): | 1245 | 33 | 5 | 9 | 1335 |

by using Newton's divided difference formula.

b. Find $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3^{rd}}$ and $\frac{3}{8^{th}}$ rule by dividing the range of integration into 6 equal parts. Hence obtain the approximate value of $\pi$ in each case. **(8)**

**Q.7** a. Use Lagrange's method to solve $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$ **(8)**

b. Use Charpit's method to find complete integral of $q = (z+px)^2$ **(8)**

**Q.8** a. A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 white and the second draw to give 4 black balls in each of the following cases-
(i) The balls are replaced before the second daw
(ii) The balls are not replaced before the second draw **(8)**

b. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. Find the chance of selecting 1 girl and 2 boys. **(8)**

**Q.9** a. A manufacturer knows that the condensers he makes contain on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensers? **(8)**

b. A sample of 100 dry battery cells tested to find the length of life produced the following results-
$\overline{x} = 12$ hours, $\sigma = 3$ hours
Assuming the data to be normally distributed, what percentage of battery cells are expected to have life-
(i) More than 15 hours
(ii) Less than 6 hours
(iii) Between 10 and 14 hours **(8)**