ROLL NO. ____________________

**Code: AE51/AC51/AT51 Subject: ENGINEERING MATHEMATICS - I**

# AMIETE – ET/CS/IT

**Time: 3 Hours** **JUNE 2014** **Max. Marks: 100**

***PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.***

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a. If z = f(y/x) then the value of $x\left(\frac{\partial z}{\partial x}\right) + y\left(\frac{\partial z}{\partial y}\right)$ is equal to

**(A)** z **(B)** -z
**(C)** 0 **(D)** 1

b. The function f(x) = $x^5 - 5x^4 + 5x^3 - 10$ has a maximum at-

**(A)** x = 0 **(B)** x = 1
**(C)** x = 2 **(D)** none of these

c. The value of the integral $\int_1^2 \int_0^x \frac{dxdy}{x^2 + y^2}$ is equal to

**(A)** $\pi \ell og 2$ **(B)** $\pi \ell og \frac{1}{2}$

**(C)** $\pi^2 \ell og 2$ **(D)** $\frac{\pi}{4} \ell og 2$

d. The rank of the matrix $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is equal to

**(A)** 1 **(B)** 2
**(C)** 3 **(D)** 4

e. The rank of the matrix A + B where $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ is equal to

**(A)** 1 **(B)** 2
**(C)** 3 **(D)** 0

f. Integrating factor of the differential equation $x\left(\frac{dy}{dx}\right) + 2y = x^2 \ell ogx$ is equal to

**(A)** $x^2$ **(B)** $\ell ogx^2$
**(C)** $\frac{1}{x}$ **(D)** $\ell ogx$

g. The orthogonal trajectories of the family of curves $y = a x^2$ is

**(A)** $x^2 + y^2 = c^2$ **(B)** $x^2 - y^2 = c^2$
**(C)** $x^2 + 2y^2 = c^2$ **(D)** $xy = c^2$

Where c is an arbitrary constant

h. The solution of the differential equation $\frac{d^2y}{dx^2}$-3 $\frac{dy}{dx}$ + 2y = $e^x$ is

**(A)** $y = c_1e^x + c_2e^{3x} + x$ **(B)** $y = (c_1 + c_2)e^x - xe^x$
**(C)** $y = c_1e^x + c_2e^{2x} - xe^x$ **(D)** $y = c_1x + c_2e^x - e^{2x}$

i. The value of the integral $\int_0^1 \frac{x^2}{\sqrt{1-x^5}}dx$ is equal to

**(A)** $\frac{1}{2}B\left(\frac{3}{5},\frac{1}{2}\right)$ **(B)** $\frac{1}{5}B\left(\frac{1}{3},\frac{1}{2}\right)$
**(C)** $B\left(\frac{3}{5},\frac{2}{5}\right)$ **(D)** $\frac{1}{5}B\left(\frac{3}{5},\frac{1}{2}\right)$

j. If $P_n(x)$ is a solution of Legendre's equation, then value of $P_0(x)$ will be

**(A)** 1 **(B)** 0
**(C)** -1 **(D)** x

**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

**Q.2** a. If $u = x\phi(y/x) + \psi(y/x)$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$ **(8)**

b. Show that the maximum and minimum of the radii vectors of the sections of the surface $(x^2 + y^2 + z^2)^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ by the plane $\lambda x + \mu y + \gamma z = 0$ are given by the equation- $\frac{a^2\lambda^2}{1 - a^2r^2} + \frac{b^2\mu^2}{1 - b^2r^2} + \frac{c^2\gamma^2}{1 - c^2r^2} = 0$ **(8)**

**Q.3** a. Evaluate $\iint (x + y)^2 dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(8)**

b. Evaluate $\iiint z^2 dxdydz$ over the sphere $x^2 + y^2 + z^2 = 1$ **(8)**

**Q.4** a. For what values of $\eta$, the equations x + y + z =1, x + 2y + 4z = $\eta$, x + 4y + 10z = $\eta^2$ have a solution and solve them completely in each case. **(8)**

b. Determine the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ **(8)**

**Q.5** a. Find by Newton – Raphson method, the real root of the equation:

$$3x = \cos x + 1$$

Nearer to 1, correct to three decimal places. **(8)**

b. Apply Runge-Kutta method of fourth order to find approximate value of y for x = 0.2, in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that y = 1 where x = 0 **(8)**

**Q.6** a. Solve the equation cos x dy = y (sin x – y) dx **(8)**

b. Find the orthogonal trajectories of the family of curves:

$$\frac{x^2}{(a^2 + \lambda)} + \frac{y^2}{(b^2 + \lambda)} = 1$$

Where $\lambda$ is the parameter. **(8)**

**Q.7** a. Find the solution of the equation $\frac{d^2y}{dx^2} + 4y = 8 \cos 2x$

given that y = 0 and $\frac{dy}{dx} = 2$ when x = 0 **(8)**

b. Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$ **(8)**

**Q.8** a. Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx = \frac{1}{(a+b)^m a^n} B(m,n)$

where B (m, n) is Beta function. **(8)**

b. Solve following differential equation $\frac{d^2y}{dx^2} - 2x^2\frac{dy}{dx} + 4xy = x^2 + 2x + 2$ in power of x. **(8)**

**Q.9** a. Prove that:

$x^2 J_n''(x) = (n^2 - n - x^2)J_n(x) + xJ_{n+1}(x)$ **(8)**

b. Prove that

$$\int_{-1}^{+1}(1-x^2)P'_m P'_n\, dx = 0$$

Where m and n are distinct positive integers. **(8)**