Code: AE101/AC101/AT101 **Subject: ENGINEERING MATHEMATICS-I**

AMIETE – ET/CS/IT *{NEW SCHEME}*

Time: 3 Hours **JUNE 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $u = \log\dfrac{x^4+y^4}{x+y}$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to

(A) 1 **(B)** 2
(C) 3 **(D)** 4

b. The rank of matrix $\begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$ is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

c. The value of the double integral $\int_0^1 \int_0^{\sqrt{1+x^2}} \dfrac{dy \cdot dx}{1+x^2+y^2}$, is

(A) $\dfrac{\pi}{4}\log\left(\sqrt{2}+1\right)$ **(B)** $\dfrac{\pi}{2}\log\left(\sqrt{3}+1\right)$

(C) $\dfrac{\pi}{3}\log\left(\sqrt{5}+1\right)$ **(D)** $\dfrac{\pi}{5}\log\left(\sqrt{2}+1\right)$

d. The smallest positive root of the equation $x^3 - 2x + 0.5 = 0$ is equal to ______, using Newton Raphson method,

(A) 1.2872 **(B)** 2.2952
(C) 3.2748 **(D)** 0.2578

e. If the differential equation $p^2 + q^2 - 1 = 0$, then dependent variable z is equal to

(A) $ax - \sqrt{1-a^2}\,y + c$ **(B)** $ax - \sqrt{1+a^2}\,y + c$

(C) $ax + \sqrt{1-a^2}\,y + c$ **(D)** $ax + \sqrt{1+a^2}\,y + c$

f. The solution of differential equation $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

(A) $y = c_1e^{3x} + c_2e^{5x}$ **(B)** $y = (c_1 + c_2)e^{3x}$
(C) $y = (c_1 - c_2)e^{5x}$ **(D)** $y = c_1e^{3x} - c_2e^{-5x}$

g. The Particular Integral (P.I.) of the differential equation $(D^2 + 6D + 9)4 = 5e^{3x}$ is equal

(A) $\frac{e^{3x}}{36}$ **(B)** $\frac{5e^{3x}}{36}$
(C) $\frac{-e^{3x}}{36}$ **(D)** $\frac{e^{-3x}}{36}$

h. $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx$ is equal to

(A) $B(m+n)/a^nb^m$ **(B)** $B(m,n)/a^nb^m$
(C) $B(mn)/a^nb^m$ **(D)** $B(m-n)/a^nb^m$

i. The value of Jacobin $\frac{\partial(u,v)}{\partial(r,\theta)}$, where $u = x^2 - y^2, v = 2xy$ and $x = r\cos\theta, u = r\sin\theta$

(A) 4 **(B)** 5
(C) 6 **(D)** 3

j. In the terms of Legendre's polynomials $\frac{1}{35}[8P_4(x) + 20P_2(x) + 7P_0(x)]$ is equal to

(A) x^3 **(B)** x^2
(C) x^4 **(D)** x^5

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If z be a homogeneous function of degree n, show that $x\frac{\partial^2 z}{\partial x \cdot \partial y} + y\frac{\partial^2 z}{\partial y^2} = (n-1)\frac{\partial z}{\partial y}$ **(8)**

b. If $z = f(x, y)$, where $x = e^u \cos v$ and $y = e^u \sin v$, then show that $y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$ **(8)**

Q.3 a. Change the order of integration and then evaluate $I = \int_0^1 \int_{x^2}^{2-x} xydxdy$ **(8)**

b. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and z = 0 **(8)**

Q.4 a. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ **(8)**

b. Determine the rank of the following matrices

(i) $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ **(8)**

Q.5 a. Find the approximate value of the root of the equation $x^3 + x - 1 = 0$ near $x = 1$, using the method of Regula-Falsi two times. **(8)**

b. Express the following system of equations in matrix form and solve them by the elimination method due to Gauss:

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10$$

(8)

Q.6 a. Solve the differential equation $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$ where $D \equiv \frac{d}{dx}$ **(8)**

b. Solve the equation, $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ **(8)**

Q.7 a. Obtain the series solution of $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$ **(8)**

b. State and prove orthogonality of Legendre polynomials. **(8)**

Q.8 a. Obtain Fourier series for the function

$$f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..........\infty=\frac{\pi^2}{8}$ **(8)**

b. Express $f(x)=x$ as a half-range cosine series in $0<x<2$ **(8)**

Q.9 a. State and prove Convolution theorem for Fourier transforms. **(8)**

b. Solve by z-transform $y_{k+1}+\frac{1}{4}yk=\left(\frac{1}{4}\right)^k\left[k\ge 0, y(0)=0\right]$ **(8)**