ROLL NO. ________________

Code: AC65 **Subject: DISCRETE STRUCTURES**

AMIETE – CS

Time: 3 Hours **JUNE 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} then

(A) A = {3, 5} B = {2, 4} **(B)** A = {4, 3} B = {5, 2}
(C) A = {4, 2} B = {3, 5} **(D)** A = {2, 4} B = {3, 5}

b. If R is symmetric relation then

(A) $R \cap R^{-1} \neq \phi$ **(B)** $R \cap R^{-1} = \phi$
(C) $R \cup R^{-1} = \phi$ **(D)** $R \cup R^{-1} = R$

c. Which of the following relations are functions over set A = {1, 2, 3, 4}:

(A) {(1, 2), (2, 3), (2, 4), (3, 4)} **(B)** {(4, 1), (3, 2), (2, 3), (1, 4)}
(C) {(4, 1), (4, 2), (4, 3), (4, 4)} **(D)** {(1, 2), (2, 3), (3, 4)}

d. If A^c is the complementary event of A, then

(A) $P(A) = P(A^c)$ **(B)** $P(A) = P(A^c) -1$
(C) $P(A) = 1 - P(A^c)$ **(D)** $P(A) = (1 - P(A))^c$

e. Every subgroup of cyclic group is

(A) Semi-group **(B)** Abelian group
(C) Quotient group **(D)** Cyclic group

f. The inverse of $\sim p \rightarrow q$ is

(A) $q \rightarrow \sim p$ **(B)** $p \rightarrow \sim q$
(C) $\sim p \rightarrow \sim q$ **(D)** $\sim q \rightarrow \sim p$

g. If p: " He is rich" and q: " he is unhappy" then choose the correct formula for the statement "He is poor or he is both rich and unhappy"

(A) $\sim p \vee (p \wedge \sim q)$ **(B)** $p \wedge (p \wedge \sim q)$
(C) $\sim p \vee (p \wedge q)$ **(D)** $p \vee (p \leftrightarrow q)$

h. The hamming distance between x = 110110 and y = 000101 is

(A) 2 **(B)** 4
(C) 7 **(D)** 5

i. A partially ordered set (poset) is called lattice if every subset of two elements has

(A) Greatest lower bound
(B) Least upper bound
(C) Both greatest lower and least upper bounds
(D) None of these

j. In a ring (R, +, *) the structure (R, *) has to be

(A) Semi-group **(B)** Monoid
(C) Group **(D)** Abelian group

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that for any two sets A and B, $A - B = A - (A \cap B)$. **(8)**

b. The probability that A hits a target is 1/3 and the probability that B hits a target is 1/5. They both fire at the target. Find the probability that: **(8)**
(i) A does not hit the target.
(ii) Both hit the target.
(iii) One of them hits the target.
(iv) Neither hits the target.

Q.3 a. Show that $B \rightarrow E$ is a valid conclusion drawn from the following premises: $A \vee (B \rightarrow D)$, $\sim C \rightarrow (D \rightarrow E)$, $A \rightarrow C$ and $\sim C$. **(8)**

b. Express the statement $(\sim(p \vee q)) \vee ((\sim p) \wedge q)$ in simplest possible form. **(8)**

Q.4 a. Show that $\neg \forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent. **(4)**

b. There are two restaurants next to each other. One has a sign that says, “Good food is not cheap” and the other has a sign that says, “Cheap food is not good”. Are the signs saying the same thing? **(4)**

c. Verify that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology. **(8)**

Q.5 a. Let A = {1, 2, 3, 4, 5, 6} and let R be the relation defined by “x divides y” written as x / y. **(8)**

(i) Write R as a set of ordered pairs.
(ii) Draw its directed graph.
(iii) Find R^{-1}.

b. Prove that $n! \geq 2^n$ for $n \geq 4$, by using the principle of mathematical induction. **(8)**

Q.6 a. Let I be the set of integers and R be a binary relation defined on set I as $R = \{(x, y) | x \equiv y \pmod 3, x \in I, y \in I\}$, show that R is an equivalence relation. **(8)**

b. Prove that if L is a bounded distributive lattice and if a complement exists in L, it is unique. **(8)**

Q.7 a. Consider the functions $f : R \rightarrow R$ and $g : R \rightarrow R$, defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 1$. Find the composite functions $(gof)(x)$ and $(fog)(x)$. **(8)**

b. Let $X = \{a, b, c\}$. Define $f : X \rightarrow X$ such that $f = \{(a, b), (b, a), (c, c)\}$. Find: **(8)**

(i) f^{-1}
(ii) f^2
(iii) f^3
(iv) f^4

Q.8 a. Show that the set of rational numbers Q forms a group under the binary operation * defined by $a * b = a + b - ab$, for all $a, b \in Q$. Is this group abelian? **(8)**

b. How many generators are there of the cyclic group of order 8? **(8)**

Q.9 a. Determine the group code (3, 6) using parity check Matrix H given by **(8)**

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b. Define Ring. Prove that if $a, b \in (R, +, \bullet)$, then $(a + b)^2 = a^2 + a\bullet b + b\bullet a + b^2$, where by x^2 we mean $x\bullet x$. **(8)**