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## DipIETE - ET/CS (NEW SCHEME)

Time: 3 Hours
JUNE 2012
Max. Marks: 100
please write your roll no. at the space provided on each page IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to $\mathbf{Q} .1$ must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the $\mathbf{Q} .1$ will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.
Q. 1 Choose the correct or the best alternative in the following:
a. The value of $\lim _{x \rightarrow 1}(x)^{\frac{1}{x-1}}$ is
(A) e
(B) 2 e
(C) $\mathrm{e}^{2}$
(D) $\mathrm{e}^{3}$
b. The value of $\int_{0}^{\pi / 2} \sin ^{7} x d x$ is
(A) $4 / 5$
(B) $35 / 16$
(C) $16 / 35$
(D) $5 / 4$
c. Amplitude of $\frac{(3-\sqrt{2} i)^{2}}{1+2 \mathrm{i}}$ is
(A) $\tan ^{-1}\left(\frac{6 \sqrt{2}-4}{12 \sqrt{2}-7}\right)$
(B) $\tan ^{-1}\left(\frac{6 \sqrt{2}+14}{12 \sqrt{2}-7}\right)$
(C) $\tan ^{-1}\left(\frac{6 \sqrt{2}+4}{12 \sqrt{2}+7}\right)$
(D) $\tan ^{-1}\left(\frac{4 \sqrt{2}+7}{5 \sqrt{2}-4}\right)$
d. If the co-ordinates of P be $(3,4,12)$ then the magnitude of $\overrightarrow{\mathrm{OP}}(\mathrm{O}$ is origin) is
(A) 15
(B) 17
(C) 11
(D) 13
e. The projection of the vector $\hat{i}-2 \hat{j}+\hat{k}$ on $4 \hat{i}-4 \hat{j}+7 \hat{k}$ is
(A) $\frac{9}{19}$
(B) $\frac{19}{9}$
(C) $\frac{11}{9}$
(D) $\frac{9}{11}$
f. What is the order and degree of the equation $\frac{d^{2} y}{d x^{2}}+a^{2} x=0$ ?
(A) Order 2, Degree 2
(B) Order 2, Degree 1
(C) Order 2, Degree 0
(D) Order 1, Degree 2
g. If $f(x)=x \sin x,(-\pi, \pi)$ then the value of $b_{n}$ is
(A) $-\pi$
(B) 0
(C) $\pi$
(D) $2 \pi$
h. value of $L\left\{\cos ^{2} 2 t\right\}$ is
(A) $\frac{1}{2}\left(\frac{1}{\mathrm{~s}}+\frac{\mathrm{s}}{\mathrm{s}^{2}+16}\right)$
(B) $\left(\frac{1}{s}+\frac{s}{s^{2}+16}\right)$
(C) $\left(\frac{1}{s}-\frac{s}{s^{2}+16}\right)$
(D) $\frac{1}{2}\left(\frac{1}{\mathrm{~s}}-\frac{\mathrm{s}}{\mathrm{s}^{2}+16}\right)$
i. value of $\mathrm{L}\left\{\mathrm{e}^{2 t} \cos ^{2} \mathrm{t}\right\}$ is
(A) $\frac{1}{2}\left\{\frac{1}{s+2}-\frac{s-2}{(s-2)^{2}+4}\right\}$
(B) $\frac{1}{2}\left\{\frac{1}{s-2}+\frac{s-2}{(s-2)^{2}+4}\right\}$
(C) $\frac{1}{2}\left\{\frac{1}{s-2}-\frac{s-2}{(s-2)^{2}+4}\right\}$
(D) 0
j. Inverse Laplace transform of $\left\{\frac{s^{2}-3 s+4}{s^{3}}\right\}$ is
(A) $1+3 t+2 t^{2}$
(B) $1-3 \mathrm{t}-2 \mathrm{t}^{2}$
(C) $1+3 \mathrm{t}-2 \mathrm{t}^{2}$
(D) $1-3 t+2 t^{2}$


## Answer any FIVE Questions out of EIGHT Questions. <br> Each question carries 16 marks.

Q. 2 a. Evaluate $\lim _{x \rightarrow 0}\left(\frac{1}{x}\right)^{2 \sin x}$
b. Expand $\log \left(1+e^{x}\right)$ in ascending powers of x as far as the term containing $\mathrm{x}^{4}$, using Maclaurin's theorem.
Q. 3 a. If $I_{n}=\int_{0}^{\pi / 2} x^{n} \sin x d x \quad, n>1$, show that $I_{n}+n(n-1) I_{n-2}=n(\pi / 2)^{n-1}$
b. The area enclosed by the hypocycloid $x^{2 / 3}+y^{2 / 3}=a^{2 / 3}$ is revolved about $x$-axis. Find the volume of the solid generated.
Q. 4 a. If $\mathrm{Z}_{1}, \mathrm{Z}_{2}$ be two complex numbers, show that
$\left|\mathrm{Z}_{1}+\mathrm{Z}_{2}\right|^{2}+\left|\mathrm{Z}_{1}-\mathrm{Z}_{2}\right|^{2}=2\left(\left|\mathrm{Z}_{1}\right|^{2}+\left|\mathrm{Z}_{2}\right|^{2}\right)$
b. If $2 \cos \theta=x+\frac{1}{x}$, prove that $2 \cos r \theta=x^{r}+\frac{1}{x^{r}}$
Q. 5 a. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices $A, B, C$ of a triangle. Show that the vector area of the triangle is $\frac{1}{2}(\vec{b} \times \vec{c}+\vec{c} \times \vec{a}+\vec{a} \times \vec{b})$
b. Find the volume of parallelopiped if $\vec{a}=-3 \hat{i}+7 \hat{j}+5 \hat{k}, \vec{b}=-3 \hat{i}+7 \hat{j}-3 \hat{k}$ and $\vec{c}=7 \hat{i}-5 \hat{j}-3 \hat{k}$ are the three co-terminous edges of the parallelopiped.
Q. 6 a. Solve $\left(D^{2}-5 D+6\right) y=e^{x} \cos 2 x$
(8)
b. Solve $\frac{d^{2} y}{d x^{2}}+9 y=\sec 3 x$
Q. 7 a. Find a Fourier series to represent $x^{2}$ in the interval ( $-l, l$ ).
(8)
b. Expand $\mathrm{f}(\mathrm{x})=\frac{1}{4}-\mathrm{x}$, if $0<\mathrm{x}<\frac{1}{2}$

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\begin{equation*}
=x-\frac{3}{4} \text {, if } \frac{1}{2}<x<1 \text { as the Fourier series of sine terms. } \tag{8}
\end{equation*}
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Q. 8 a. Find the Laplace transform of $\sin 2 t \cos 3 t$
b. Find Laplace transform of $\frac{\cos a t-\cos b t}{t}$
Q. 9 a. Evaluate $L^{-1}\left\{\frac{s^{2}}{\left(s^{2}+a^{2}\right)^{2}}\right\}$
b. Apply convolution theorem to solve $\mathrm{L}^{-1}\left\{\frac{1}{\left(\mathrm{~s}^{2}+1\right)\left(\mathrm{s}^{2}+9\right)}\right\}$

