

DiplETE – ET/CS (NEW SCHEME)

Time: 3 Hours

JUNE 2012

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of $\lim_{x \rightarrow 1} (x)^{\frac{1}{x-1}}$ is

- (A) e (B) 2e
(C) e² (D) e³

b. The value of $\int_0^{\pi/2} \sin^7 x \, dx$ is

- (A) 4/5 (B) 35/16
(C) 16/35 (D) 5/4

c. Amplitude of $\frac{(3 - \sqrt{2}i)^2}{1 + 2i}$ is

- (A) $\tan^{-1}\left(\frac{6\sqrt{2} - 4}{12\sqrt{2} - 7}\right)$ (B) $\tan^{-1}\left(\frac{6\sqrt{2} + 14}{12\sqrt{2} - 7}\right)$
(C) $\tan^{-1}\left(\frac{6\sqrt{2} + 4}{12\sqrt{2} + 7}\right)$ (D) $\tan^{-1}\left(\frac{4\sqrt{2} + 7}{5\sqrt{2} - 4}\right)$

d. If the co-ordinates of P be (3, 4, 12) then the magnitude of \overrightarrow{OP} (O is origin) is

- (A) 15 (B) 17
(C) 11 (D) 13

e. The projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on $4\hat{i} - 4\hat{j} + 7\hat{k}$ is

- (A) $\frac{9}{19}$ (B) $\frac{19}{9}$
(C) $\frac{11}{9}$ (D) $\frac{9}{11}$

- b. The area enclosed by the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$ is revolved about x-axis. Find the volume of the solid generated. (8)
- Q.4** a. If Z_1, Z_2 be two complex numbers, show that $|Z_1 + Z_2|^2 + |Z_1 - Z_2|^2 = 2(|Z_1|^2 + |Z_2|^2)$ (8)
- b. If $2 \cos \theta = x + \frac{1}{x}$, prove that $2 \cos r\theta = x^r + \frac{1}{x^r}$ (8)
- Q.5** a. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A,B,C of a triangle. Show that the vector area of the triangle is $\frac{1}{2}(\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b})$ (8)
- b. Find the volume of parallelopiped if $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $\vec{b} = -3\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ are the three co-terminous edges of the parallelopiped. (8)
- Q.6** a. Solve $(D^2 - 5D + 6)y = e^x \cos 2x$ (8)
- b. Solve $\frac{d^2 y}{dx^2} + 9y = \sec 3x$ (8)
- Q.7** a. Find a Fourier series to represent x^2 in the interval $(-l, l)$. (8)
- b. Expand $f(x) = \frac{1}{4} - x$, if $0 < x < \frac{1}{2}$
 $= x - \frac{3}{4}$, if $\frac{1}{2} < x < 1$ as the Fourier series of sine terms. (8)
- Q.8** a. Find the Laplace transform of $\sin 2t \cos 3t$ (8)
- b. Find Laplace transform of $\frac{\cos at - \cos bt}{t}$ (8)
- Q.9** a. Evaluate $L^{-1} \left\{ \frac{s^2}{(s^2 + a^2)^2} \right\}$ (8)
- b. Apply convolution theorem to solve $L^{-1} \left\{ \frac{1}{(s^2 + 1)(s^2 + 9)} \right\}$ (8)