ROLL NO. _____

Code: DE51/DC51 Subject: ENGINEERING MATHEMATICS - I

DipIETE – ET/CS (NEW SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. $\lim_{x\to 1} \frac{x-1}{\log x}$ is :

(A) 1 **(B)** $\frac{1}{2}$

(C) 2 **(D)** -1

b. The centroid of the triangle with vertices (2, 7), (3, 4) and (–6, 4) is

(A) $\left(5,\frac{1}{3}\right)$ **(B)** $\left(\frac{1}{3},-5\right)$

(C) $\left(-\frac{1}{3},5\right)$ **(D)** $\left(-5,\frac{1}{3}\right)$

c. $\int \sin^3 x dx$ is

(A) $\frac{3}{4}\sin x+\frac{1}{12}\sin 3x+C$ **(B)** $-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x+C$

(C) $\frac{3}{4}\sin x+\frac{1}{12}\cos 3x+C$ **(D)** $-\frac{3}{4}\cos x-\frac{1}{12}\cos 3x+C$

d. If $\Delta=\begin{vmatrix}\omega & \omega^2 & 1\\ 1 & \omega & \omega^2\\ \omega^2 & 1 & \omega\end{vmatrix}$, then the value of Δ is

(A) -1 **(B)** 1

(C) 2 **(D)** 0

e. If $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, then x, y, z, w is equal to

(A) 1, 2, 3, 4 **(B)** 2, 4, 1, 3
(C) –1, 3, 2, 4 **(D)** 1, –2, 1, 4

f. The order and degree of differential equation $\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}$ is

(A) O = 2, D = 1 **(B)** O = 1, D = 1
(C) O = 2, D = 3 **(D)** O = 2, D = 2

g. The middle term in the expansion of $\left(x + \frac{1}{x}\right)^{12}$ is

(A) 1001 **(B)** 923
(C) 1004 **(D)** 924

h. The value of $2\cos\left(\frac{\pi}{4} + \theta\right)\cos\left(\frac{\pi}{4} - \theta\right)$ is

(A) $\sin 2\theta$ **(B)** $\cos 2\theta$
(C) $\sin\theta\cos\theta$ **(D)** $\cos 3\theta$

i. The distance between the pair of points $A\left(am_1^2, 2am_1\right), B\left(am_2^2, 2am_2\right)$ is

(A) $a(m_2 - m_1)\sqrt{(m_2 + m_1)^2 + 4}$ **(B)** $a(m_2 + m_1)\sqrt{(m_2 - m_1)^2 + 4}$
(C) $a(m_1 + m_2)\sqrt{(m_1 + m_2)^2 + 4}$ **(D)** $a(m_2 - m_1)\sqrt{(m_2 + m_1)^2 - 4}$

j. If $y = \log(\sec x + \tan x)$, then $\frac{dy}{dx}$ is

(A) sec x cosec x **(B)** tan x
(C) sec x **(D)** sec x tan x

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$. **(8)**

b. Find all the points of maxima minima and the corresponding maximum and minimum values of the function $f(x) = -x^3 + 12x^2 - 5$. **(8)**

Q.3 a. Evaluate $\int \frac{\sec^2 x}{5\tan^2 x - 12\tan x + 14} dx$ **(8)**

b. Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$ **(8)**

Q.4 a. Solve the following equation $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$ **(8)**

b. Solve with the help of matrices the simultaneous equations:

$x + y + z = 3$

$x + 2y + 3z = 4$ **(8)**

$x + 4y + 9z = 6$

Q.5 a. Solve $\frac{dy}{dx} = \cos^3 x \sin^4 x + x\sqrt{2x+1}$ **(8)**

b. Solve $\frac{dy}{dx} + y \sec x = \tan x$ **(8)**

Q.6 a. Find the term independent of x in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ **(8)**

b. If the first term of an AP is 2 and the sum of first five terms is equal to one fourth of the sum of the next five terms, find the sum of first 30 terms. **(8)**

Q.7 a. Prove that $\cos 20^\circ \cos 30^\circ \cos 40^\circ \cos 80^\circ = \frac{\sqrt{3}}{16}$ **(8)**

b. If $A + B + C = \pi$, show that $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ **(8)**

Q.8 a. Find the equation of a line passing through the point (2, 3) and making an angle of 45° with the line $3x + y - 5 = 0$. **(8)**

b. If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ **(8)**

Q.9 a. Find the equation of the circle passing through the point (2, 4) & has its centre at the intersection of lines $x - y = 4$ and $2x + 3y = -7$. **(8)**

b. Show that $4x^2 + 16y^2 - 24x - 32y - 12 = 0$ is the equation of an ellipse. Find its vertices, foci, eccentricity, directrices, major axis, minor and latusrectum. **(8)**