ROLL NO. ___________________

Code: AE57/AC57/AT57 Subject: SIGNALS AND SYSTEMS

AMIETE – ET/CS/IT (NEW SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty}\delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

(A) 1 **(B)** -1
(C) 0 **(D)** $\pi/2$

b. A system with input $x[n]$ and output $y[n]$ is given as $y[n]=\left[\sin\frac{5}{6}\pi n\right]x[n]$

(A) linear, stable and invertible **(B)** non-linear, stable and non-invertible
(C) linear, stable and non-invertible **(D)** linear, unstable and invertible

c. Convolution of $x(t+5)$ with impulse function $\delta(t-7)$ is equal to:

(A) $x(t-12)$ **(B)** $x(t+12)$
(C) $x(t-2)$ **(D)** $x(t+2)$

d. If the impulse response of discrete time system is $h[n]=-5^n u[-n-1]$, then the system function $H[z]$ is equal to

(A) $\frac{-z}{z-5}$ and ROC $|Z|>5$ **(B)** $\frac{z}{z-5}$ and ROC $|Z|<5$
(C) $\frac{-z}{z-5}$ and ROC $|Z|<5$ **(D)** $\frac{z}{z-5}$ and ROC $|Z|>5$

e. The fourier transform of a real valued time signal has

(A) odd symmetry **(B)** even symmetry
(C) conjugate symmetry **(D)** no symmetry

f. The fourier series representation of an impulse train denoted by: $\delta(t-nT_o)$ is given by

(A) $\frac{1}{T_o}\sum_{n=-\infty}^{\infty}\exp\left(-j\frac{2\pi nt}{T_o}\right)$ **(B)** $\frac{1}{T_o}\sum_{n=-\infty}^{\infty}\exp\left(-j\frac{\pi nt}{T_o}\right)$
(C) $\frac{1}{T_o}\sum_{n=-\infty}^{\infty}\exp\left(j\frac{\pi nt}{T_o}\right)$ **(D)** $\frac{1}{T_o}\sum_{n=-\infty}^{\infty}\exp\left(j\frac{2\pi nt}{T_o}\right)$

g. Laplace transform of signal $x(t) = e^{-at}u(t)$ for a > 0 is

(A) $\frac{1}{s+a}$ **(B)** $\frac{s}{s+a}$

(C) $\frac{s}{(s+a)^2}$ **(D)** $\frac{a}{s+a}$

h. $x[n] = a^{|n|}, |a| < 1$, the discrete time fourier transform is given by

(A) $\frac{1-a^2}{1-2a\sin\Omega + a^2}$ **(B)** $\frac{1-a^2}{1-2a\cos\Omega + a^2}$

(C) $\frac{1-a^2}{1-2ja\sin\Omega + a^2}$ **(D)** None of these

i. Given the z-transform $X(z) = \frac{z(8z-7)}{4z^2 - 7z + 3}$. Given x(n) is causal, then $x[\infty]$ is

(A) 1 **(B)** 2

(C) ∞ **(D)** 0

j. The signal $x(t) = e^{j(2t + \pi/4)}$ is a

(A) power signal with $P_\infty = 1$ **(B)** power signal with $P_\infty = 2$

(C) energy signal with $E_\infty = 2$ **(D)** energy signal with $E_\infty = 1$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Compute the output y(t) for a continuous – time LTI system whose impulse response h(t) and the input x(t) are given by $h(t) = e^{-\alpha t}u(t)$ $x(t) = e^{\alpha t}u(-t)\ \alpha > 0$. **(8)**

b. Determine the response y(n), $n \geq 0$, of the system by second order difference equation, $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ for the input $x(n) = 4^n u(n)$ **(8)**

Q.3 a. Consider the periodic sequence $x[n]$ shown in Fig.1. Determine the fourier coefficient c_k and sketch the magnitude spectrum $|c_k|$. **(8)**

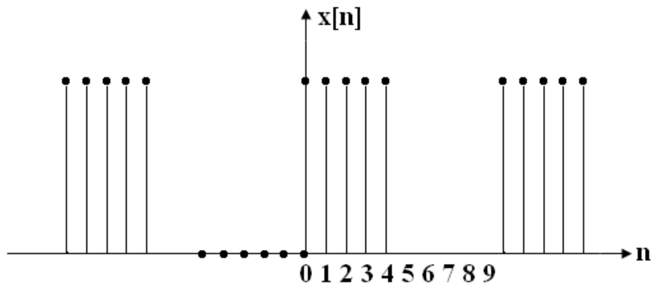

Fig.1

b. Find the cosine representation fourier series for the periodic signal shown in Fig.2 **(8)**

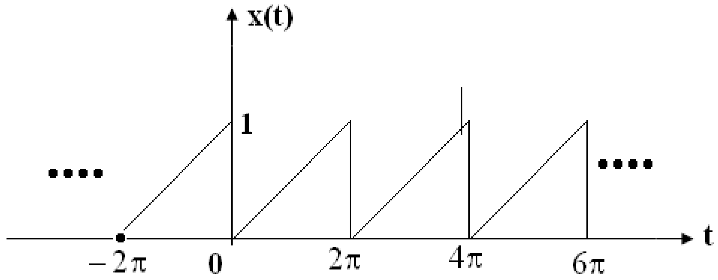

Fig.2

Q.4 a. Consider a continuous time LTI system described by $\frac{dy(t)}{dt} + 2y(t) = x(t)$. Using the fourier transform, find the output y(t) to each of the following input signals:

(i) $x(t) = e^{-t}u(t)$ (ii) $x(t) = u(t)$ **(8)**

b. (i) Verify the integration property, that is $\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \pi X(0)\delta(\omega) + \frac{1}{j\omega}X(\omega)$

(ii) Prove the frequency convolution theorem, that is

$$x_1(t)x_2(t) \leftrightarrow \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$$ **(8)**

Q.5 a. Consider a casual LTI system described by the difference equation $y[n] + \frac{1}{2}y[n-1] = x[n]$

(i) Determine the frequency response $H(j\Omega)$ of the system.

(ii) What is the response of the system to the following input

$$x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$$ **(8)**

b. Consider a system consisting of the cascade of two LTI system with frequency responses,

$$H_1(e^{j\omega}) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}, \text{ and, } H_2(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-j2\omega}}$$

Find the difference equation describing the overall system. **(8)**

Q.6 a. The frequency response of a causal and stable continuous time LTI system is expressed as $H(j\omega) = \frac{1 - j\omega}{1 + j\omega}$

(i) Determine the magnitude of $H(j\omega)$.

(ii) Find which of the following statements is true about $\tau(\omega)$, the group delay of the system

(a) $\tau(\omega) = 0$, for $\omega > 0$

(b) $\tau(\omega) > 0$, for $\omega > 0$

(c) $\tau(\omega) < 0$, for $\omega > 0$ **(8)**

b. Discuss the following:
(i) first order sample-hold circuit
(ii) reconstruction of analog signal from the sampled version using a low-pass filter. **(8)**

Q.7 a. Find the inverse laplace transform of the following:

(i) $X(s) = \dfrac{s}{s^2 + 5s + 6}$ (ii) $X(s) = \dfrac{3s^2 + 8s + 6}{(s+2)(s^2 + 2s + 1)}$ **(8)**

b. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by $y''(t) + y'(t) - 2y(t) = x(t)$
(i) Find the system function H(s).
(ii) Determine the impulse response $h(t)$ for each of the following 3 cases
(a) the system is causal.
(b) the system is stable.
(c) the system is neither causal nor stable. **(8)**

Q.8 a. Find the inverse z-transform of

(i) $X(z) = \dfrac{1}{(1 - az^{-1})^2}$ $|z| < |a|$

(ii) $X(z) = \dfrac{\frac{1}{4} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{1}{4} z^{-1}\right)}$ ROC: $|z| > \frac{1}{2}$ **(8)**

b. A casual system is represented by the following difference equation:
$$y(n) + \frac{1}{4} y(n-1) = x(n) + \frac{1}{2} x(n-1)$$
(i) Find the system function H(z) and give the corresponding region of convergence.
(ii) Find the unit sample response of the system.
(iii) Find the frequency response $H(e^{j\omega})$ and determine its magnitude and phase. **(8)**

Q.9 a. Consider the random process $X(t) = A\cos(\omega_o t + \phi)$ where A and ω_o are constants and ϕ is random variable distributed on $[-\pi, \pi]$. Check when X(t) is Ergodic? **(8)**

b. For the linear time invariant system shown in Fig.3, if the autocorrelation $R_x(e)$ of random process X(t) and impulse response $h(\tau)$ is expressed by

$R_X(\tau) = e^{-\alpha|\tau|}$ and
$h(\tau) = e^{-\beta\tau} u|\tau|$

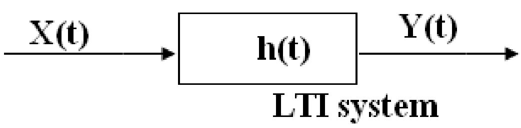

Fig.3

Where, α and β are constants and $u(\tau)$ is unit step function. Then, find the spectral density function of output process Y(t). **(8)**