ROLL NO. ___________________

Code: AE51/AC51/AT51 Subject: ENGINEERING MATHEMATICS - I

AMIETE – ET/CS/IT (NEW SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If x=u(1–v) and y=uv, then $\frac{\delta(x,y)}{\delta(u,v)}$ is equal to

(A) u **(B)** v
(C) 1 **(D)** 0

b. The value of $\int_0^1 \int_0^{1-x} dxdy$ is

(A) 1 **(B)** $\frac{1}{2}$
(C) $\frac{1}{3}$ **(D)** $\frac{1}{4}$

c. The value of K for which equations 3x+y–Kz=0, 4x–2y–3z=0 and 2Kx+4y+Kz=0 are consistent, is

(A) 4 **(B)** 3
(C) 2 **(D)** 1

d. The order of convergence in Newton-Raphson method is

(A) 1 **(B)** 1.6
(C) 2 **(D)** 2.4

e. The equation $(2x^3y^2+x^4)dx+(x^4y+y^4)dy=0$

(A) variable separable **(B)** Homogeneous
(C) Linear **(D)** Exact

f. The solution of $\frac{d^2y}{dx^2} + 3a\frac{dy}{dx} - 4a^2y = 0$ is

(A) $y=C_1e^{ax}+C_2e^{4ax}$ **(B)** $y=C_1e^{-ax}+C_2e^{-4ax}$
(C) $y=C_1e^{ax}+C_2e^{-4ax}$ **(D)** $y=C_1e^{-ax}+C_2e^{4ax}$

g. When X(x) is any function of x, $\frac{1}{D-a}X(x)$ is equal to

(A) $e^{ax}\int X(x)e^{-ax}dx$ **(B)** $e^{ax}\int X(x)e^{ax}dx$

(C) $e^{-ax}\int X(x)e^{ax}dx$ **(D)** $e^{-ax}\int X(x)e^{-ax}dx$

h. $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ is equal to

(A) $\sqrt{\pi}$ **(B)** π

(C) $\pi^{\frac{3}{2}}$ **(D)** None of these

i. $J_{½}(x)$ is equal to

(A) $J_{-\frac{1}{2}}(x)\sin x$ **(B)** $J_{-\frac{1}{2}}(x)\cos x$

(C) $J_{-\frac{1}{2}}(x)\tan x$ **(D)** $J_{-\frac{1}{2}}(x)\cot x$

j. The polynomial $2x^2+x+3$ in terms of Legendre polynomials is

(A) $\frac{1}{3}(4P_2 - 3P_1 + 11P_0)$ **(B)** $\frac{1}{3}(4P_2 + 3P_1 + 11P_0)$

(C) $\frac{1}{3}(4P_2 + 3P_1 - 11P_0)$ **(D)** $\frac{1}{3}(4P_2 - 3P_1 - 11P_0)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $u=\log(x^3+y^3+z^3-3xyz)$, show that $\left(\frac{\delta}{\delta x}+\frac{\delta}{\delta y}+\frac{\delta}{\delta z}\right)^2 u = -9(x+y+z)^{-2}$ **(8)**

b. Expand f(x,y)=sin(xy) in powers of (x–1) and $\left(y-\frac{\pi}{2}\right)$ up to the second degree terms. **(8)**

Q.3 a. Evaluate by changing the order of integration of $\int_0^\infty \int_0^x x e^{-\frac{x^2}{y}} dy dx$ **(4+4)**

b. Find the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$ **(8)**

Q.4 a. Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ **(8)**

b. For what values of K, the equations x+y+z=1, 2x+y+4z=K, $4x+y+10z=K^2$ have a solution and solve them completely in each case. **(8)**

Q.5 a. Use Gauss-Seidal method to solve the equations

$$10x + 2y + z = 9$$
$$2x + 20y - 2z = -44$$
$$-2x + 3y + 10z = 22$$ **(8)**

b. Employ Taylor's series method to obtain an approximate value of y at x=0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$. Compare the numerical solution obtained with the exact solution. **(6+2)**

Q.6 a. Solve the differential equation $ye^y dx = (y^3 + 2xe^y)dy$ **(8)**

b. Find the orthogonal trajectories of the family of coaxial circles $x^2+y^2+2\lambda y+c=2$, λ being a parameter. **(8)**

Q.7 a. Solve the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sin 2x$ **(8)**

b. Solve the simultaneous equations

$$\frac{dx}{dt} + y = \sin t$$
$$\frac{dy}{dt} + x = \cos t$$

Given that x=2 and y=0 when t=0 **(8)**

Q.8 a. Show that $\beta(m,m) = \dfrac{\sqrt{\pi}\,\Gamma(m)}{2^{2m-1}\,\Gamma\left(m+\frac{1}{2}\right)}$ **(8)**

b. Obtain the series solution of $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 0$ **(8)**

Q.9 a. Show that $\displaystyle\int_{-1}^{+1} P_m(x)P_n(x)dx = \begin{cases} 0, & m \neq n \\ \dfrac{2}{2n+1}, & m = n \end{cases}$ **(4+4)**

b. Prove that

$$\frac{d}{dx}\left\{J_n^2(x) + J^2{}_{n+1}(x)\right\} = 2\left\{\frac{n}{x}J_n^2(x) - \frac{n+1}{x}J_{n+1}^2(x)\right\}$$ **(8)**