

Code: AE07

Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING

- f. LU decomposition is guaranteed when the coefficient matrix A is positive definite. This condition is
- (A) necessary only (B) sufficient only
 (C) both necessary & sufficient (D) none of these
- g. Consider the following statements:
 (i) In interpolation, Lagrange and divided difference polynomials are two different forms of the same polynomial.
 (ii) When the degree of interpolating polynomial is fixed, a judicious choice of the initial points may improve the result.
 Which of the following statements are correct?
- (A) (i) only (B) (ii) only
 (C) Both (i) & (ii) (D) None of these
- h. Consider the following statements:
 (i) In numerical differentiation methods, the round-off error is always proportional to some power of step length h.
 (ii) In numerical differentiation methods, the truncation error is always proportional to some power of step length h.
 Which of the following statements are correct?
- (A) (i) only (B) (ii) only
 (C) Both (i) & (ii) (D) None of these
- i. If the formula $\int_0^h f(x) dx = h \left[af(0) + bf\left(\frac{h}{3}\right) + cf(h) \right]$ is exact for polynomials of as high order as possible, the values of a, b and c are given as respectively:
- (A) $0, \frac{3}{4}, \frac{1}{4}$ (B) $1, \frac{3}{4}, \frac{1}{4}$
 (C) $0, \frac{1}{4}, \frac{1}{4}$ (D) $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$
- j. The value of $y(0.2)$ using Euler's method, given that $y' = x(y + x) - 2$, $y(0) = 2$ ($h = 0.2$) is
- (A) 1.4 (B) 1.6
 (C) 1.272 (D) None of these

**Answer any FIVE Questions out of EIGHT Questions.
 Each question carries 16 marks.**

- Q.2** a. Write a program in C to evaluate $\int_0^1 \frac{x}{(1+x^2)} dx$ using Trapezoidal rule, using 10 subintervals. (8)
- b. Find the iterative methods based on the Newton-Raphson method for finding the smallest positive root of $x^4 - x - 10 = 0$, correct to three decimal places. (8)
- Q.3** a. Given equation $x - e^x = 0$. Use secant method to find the smallest positive root of the given equation correct to three decimal places. (8)
- b. What is the use of malloc() function? How can it be used in a C program? (8)
- Q.4** a. Solve the system of equations by LU decomposition
- $$\begin{aligned} 4x_1 + x_2 + x_3 &= 4 \\ x_1 + 4x_2 - 2x_3 &= 4 \\ 3x_1 + 2x_2 - 4x_3 &= 6 \end{aligned} \quad (8)$$
- b. Solve the following system of equations by Jacobi's method (perform 3 iterations)
- $$\begin{aligned} 4x + y + 2z &= 4 \\ 3x + 5y + z &= 7 \\ x + y + 3z &= 3 \end{aligned} \quad (8)$$
- Q.5** a. Obtain a second degree polynomial approximation to $f(x) = (1+x)^{\frac{1}{2}}, x \in [0, 0.1]$, using the Taylor series expansion about $x = 0$. Use the expansion to approximate $f(0.05)$ and find a bound of the Truncation Error. (8)
- b. For the following data, calculate the divided differences and obtain the divided difference polynomial. Interpolate at $x = 3.5$. (8)
- | | | | | | | |
|---------|---|----|----|---|---|----|
| $x:$ | 0 | 1 | 2 | 4 | 5 | 6 |
| $f(x):$ | 1 | 14 | 15 | 5 | 6 | 19 |
- Q.6** a. Determine the linear least square approximation to the function $y = 2^x$ at the points $x_i = 0, 1, 2, 3, 4$. (8)
- b. Let $f(x) = \ln(1+x), x_0 = 1$ and $x_1 = 1.1$. Find an approximate value of $f(1.04)$ by Lagrange interpolation and obtain a bound on the truncation error. (8)
- Q.7** a. Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using composite Simpson's rule taking eight intervals. Also obtain the number of function evaluations required to get an accuracy of 10^{-6} when the integral is evaluated directly by using Simpson's rule. (8)

- b. Evaluate the integral $I = \int_0^{\pi} e^x \cos x \, dx$ using Gauss-Legendre three point formula. (8)

- Q.8** a. By use of repeated Richardson extrapolation, find $f'(1.0)$ from the following values:

x	f(x)
0.6	0.707178
0.8	0.859892
0.9	0.925863
1.0	0.984007
1.1	1.033743
1.2	1.074575
1.4	1.127986

- Apply the approximate formula $f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$ with $h = 0.4, 0.2, 0.1$. (8)

- b. Given $\frac{dy}{dx} = \frac{1}{x+y}$, where $y(0) = 1$, find $y(0.5)$ and $y(0.1)$ using Runge-Kutta fourth order method. (8)

- Q.9** a. Using Gauss-Jordan method with partial pivoting, solve the system of equations

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 4 & -2 & 1 & 5 \\ 3 & -1 & 3 & 8 \end{array} \right]. \quad (8)$$

- b. Compute the number of steps required in Bisection method, given the initial guess values are a and b. (8)