ROLL NO. ____________________

Code: AE01/AC01/AT01 Subject: MATHEMATICS-I

AMIETE – ET/CS/IT (OLD SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. $\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$ is equal to

(A) 0 **(B)** 1
(C) ∞ **(D)** does not exist

b. If $u = \frac{y^3 - x^3}{y^2 + x^2}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(A) x **(B)** y
(C) u **(D)** none of these

c. The value of the integral $\iint dxdy$ over the triangle with vertices (0,0), (2, 0) (0,2) is

(A) 1 **(B)** 2
(C) 3 **(D)** none of these

d. $(x^3 + y)dx + (ax + by^3)dy = 0$ is exact if

(A) a = 1 **(B)** b = 2
(C) a = 2, b = 3 **(D)** None of these

e. The general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = e^x$ is

(A) $y = c_1e^{-x} + c_2e^{-3x} - \frac{1}{4}e^x$ **(B)** $y = c_1e^{-x} + c_2e^{3x} - \frac{1}{4}e^x$

(C) $y = c_1e^{-x} + c_2e^{-3x} + \frac{1}{4}e^x$ **(D)** $y = c_1e^{-x} + c_2e^{3x} + \frac{1}{4}e^x$

f. The rank of the matrix $\begin{bmatrix} 1 & -1 & 3 & 3 \\ 1 & 4 & -2 & -1 \\ 3 & 2 & 4 & 5 \end{bmatrix}$ is

(A) 2 **(B)** 3
(C) 4 **(D)** none of these

g. If two eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 0 and 3, the third eigen value is

(A) 15 **(B)** 10
(C) 11 **(D)** None of these

h. $P_2(1.5)$ is equal to [where $P_2(x)$ represent Legendre polynomials for n = 2]
(A) 2.25 **(B)** 2.675
(C) 2.875 **(D)** None of these

i. $J_2(x)$ is equal to

(A) $\frac{2}{x}J_0(x) - J_1(x)$ **(B)** $\frac{2}{x}J_1(x) - J_0(x)$

(C) $J_1(x) - \frac{2}{x}J_0(x)$ **(D)** None of these

j. The total differential of the function z = x+y+xy at the point (1, 2) for $\Delta x = 0.1$ and $\Delta y = -0.2$, is
(A) 0.1 **(B)** 0.2
(C) -0.1 **(D)** -0.2

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. If $u = f(x, y)$ and x and y are functions of t, show that $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$ **(8)**

b. If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right), x > 0, y > 0$, evaluate

$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ **(8)**

Q.3 a. Obtain Taylor's series expansion of $f(x, y) = \tan^{-1}\frac{y}{x}$ in powers of $(x-1)$ and $(y-1)$ upto second degree terms. **(8)**

b. Change the order of integration and then evaluate $\int_0^1 \int_y^{y^{1/3}} e^{x^2}\,dx\,dy$ **(2+6)**

Q.4 a. Find the volume of the solid which is bounded by the cylinder $x^2 + y^2 = 1$ and the planes y + z = 1 and z = 0. **(8)**

b. Solve the differential equation $x\,dx + y\,dy + 2(x^2 + y^2)dy = 0$ **(8)**

Q.5 a. Use method of variation of parameters to solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x}$ **(8)**

b. Solve the differential equation $\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$ **(8)**

Q.6 a. Solve the differential equation $x^3\frac{d^3y}{dx^3} + 5x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + y = x^2 + \log x$ **(8)**

b. If A is a non-singular matrix and if $I + A + A^2 + A^3 + ... + A^n = 0$. Show that $A^{-1} = A^n$. **(8)**

Q.7 a. Use elementary row transformations to find inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ **(8)**

b. If the system of equations $x + ay + az = 0$, $bx + y + bz = 0$, $cx + cy + z = 0$ where a,b,c are non-zero and non-unity, has a non-trivial solution, show that $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = -1$ **(8)**

Q.8 a. Find eigen values and eigen vectors of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ **(8)**

b. Find the series solution, about x = 0, of the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ **(8)**

Q.9 a. Show that $4J_0'''(x)+3J_0'(x)+J_3(x)=0$ **(8)**

b. Prove that $\frac{1}{\sqrt{1-2xt+t^2}}=\sum_{n=0}^{\infty}t^nP_n(x), t\neq 1$ **(8)**