ROLL NO. ____________________

Code: AC09/AT09 **Subject: NUMERICAL COMPUTING**

AMIETE – CS/IT (OLD SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 $\times$ 10)

a. The error quantity which must be added to the finite representation of a computed number in order to make it the true representation of that number is called

(A) truncation error **(B)** round-off error
(C) relative error **(D)** absolute error

b. The iterative method which requires more than one evaluations for each iteration is

(A) Bisection method **(B)** Secant method
(C) Regula-Falsi method **(D)** Newton Raphson method

c. LU decomposition method involves the number of unknowns given by

(A) $n^2 + n$ **(B)** $(n^2 + n)/2$
(C) n^2 **(D)** None of these

d. Consider the following statements:
(i) In Jacobi iteration method, at the end of each iteration, the complete vector $x^{(k)}$ is replaced for obtaining the solution.
(ii) In Gauss-Seidal method, the vector $x^{(k)}$ is replaced element by element in each iteration for obtaining the solution.
Which of the following statements are correct?

(A) (i) only **(B)** (ii) only
(C) Both (i) & (ii) **(D)** None of these

e. The eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ are

(A) 1,1,1 **(B)** 1,2,–1
(C) 1,2,–2 **(D)** 1,2,2

f. The divided difference $f[x_0, x_1, x_2]$ is equal to

(A) $\Delta^2 f_o$ **(B)** $\Delta^2 f_o / h^2$
(C) $\Delta^2 f_o / 2h^2$ **(D)** $\Delta^2 f_o / 2h$

g. The following data is given:

$$\begin{array}{cccccc} x: & -2 & -1 & 0 & 1 & 2 \\ f(x): & 15 & 1 & 1 & 3 & 19 \end{array}$$

The least square linear polynomial approximation to the above data is

(A) $7.8 + x$ **(B)** $7.8 + 2x$
(C) $7.8 - x$ **(D)** $7.8 - 2x$

h. The error which is already present in the statement of the problem before its solution is called

(A) Relative Error **(B)** Absolute error
(C) Internet Error **(D)** None of these

i. The approximate value of $I = \int_0^1 \frac{\sin x}{x} dx$ using two-point rule is given by

(A) 0.9589 **(B)** 0.9546
(C) 0.9550 **(D)** 0.9555

j. Which of the following numerical method use a weighted average of slopes on the given interval, instead of a single slope?

(A) Euler method **(B)** Runge-Kutta method
(C) Taylor series method **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the iterative methods based on the Newton-Raphson method for finding the cube roots of N, where N is a positive real number. Hence, apply the method to N = 18 to obtain the result correct to three decimal places. **(8)**

b. Given the polynomial equation $x^3 - x - 4 = 0$.
(i) Find an interval in which the smallest positive root lies.
(ii) Using this interval, perform three iterations of bisection method.
(iii) Taking any two approximations in the interval obtained in (ii), perform one iteration of the secant method. **(8)**

Q.3 a. Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

Using Crout's method. **(8)**

b. Solve the following system of equations by Jacobi's iterative method (perform 3 iterations)

$$\begin{aligned} 6x_1 - 2x_2 + x_3 &= 11 \\ -2x_1 + 7x_2 + 2x_3 &= 5 \\ x_1 + 2x_2 - 5x_3 &= -1 \end{aligned}$$ **(8)**

Q.4 a. Using the Jacobi method, find all the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$$ **(8)**

b. Find the smallest eigenvalue in magnitude of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

using four iterations of the inverse power method with the initial approximation as $(1, \ 1, \ 1)^T$. **(8)**

Q.5 a. The following data for the function $f(x)=x^4$ is given

x	0.4	0.6	0.8
f(x)	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation. **(8)**

b. Determine an appropriate step size to use, in the construction of a table of $f(x) = (1+x)^6$ on [0, 1]. The truncation error for linear interpolation is to be bounded by 5×10^{-5}. **(8)**

Q.6 a. For the method $f'(x_0) = \dfrac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \dfrac{h^2}{3} f'''(\xi), \quad x_0 < \xi < x_2$

determine the optimal value of h, using the criteria

(i) $|RE| = |TE|$

(ii) $|RE| + |TE|$ = minimum

Also obtain the minimum total error. **(8)**

b. Obtain the least square polynomial of degree 2 for $f(x) = x^{1/2}$ on [0, 1]. **(8)**

Q.7 a. Construct divided difference table for the function $f(x) = 1/x^2$,

where x = a, b, c, d. **(8)**

b. Compute $f''(0.6)$ from the following table:

x	f(x)
0.2	1.420072
0.4	1.881243
0.5	2.128147
0.6	2.386761
0.7	2.657971
0.8	2.942897
1.0	3.559753

by using the approximate formula $f''(x_0) = \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$ with h = 0.4, 0.2 and 0.1 and perform repeated Richardson's extrapolation. **(8)**

Q.8 a. Evaluate the integral $I = \int_{0.2}^{1.5} e^{-x^2} dx$ using the three point Gauss-Legendre formula. **(8)**

b. Compute the integral $I = \int_0^1 \frac{x\,dx}{x^3+10}$ using trapezoidal rule with the number of points 3, 5 and 9. Improve the results using Romberg integration. **(8)**

Q.9 a. For the equation $\frac{dy}{dx} = -2x - y$, y(0) = –1 with h = 0.1, find the value of y when x = 0.1 and x = 0.2 using Runge-Kutta fourth order method. **(8)**

b. Consider the initial value problem $y' = x(y+x) - 2, y(0) = 2$. Use Euler's method with step sizes h = 0.15 and h = 0.2 to compute approximations to y(0.6), up to 5 places of decimal. **(8)**