

DiplETE – ET/CS (Current & New Scheme)

Time: 3 Hours

June 2019

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of $\lim_{x \rightarrow 0} \frac{\sin^2 x/3}{x^2}$ is

(A) $\frac{2}{9}$

(B) $\frac{4}{9}$

(C) $\frac{1}{9}$

(D) 0

b. If $y = \sin(x+a)\cos(x+a)$, then $\frac{dy}{dx}$ is

(A) $-\cos(x+a)$

(B) $\sin(x+a)$

(C) $\sin 2(x+a)$

(D) $\cos 2(x+a)$

c. The value of $\int \sin(ax+b) dx$ is

(A) $-\frac{1}{a}\cos(ax+b)+C$

(B) $\frac{1}{a}\cos(ax+b)+C$

(C) $\cos(ax+b)+C$

(D) $-\cos(ax+b)+C$

d. The order and degree of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^3 = a^2 \left(\frac{d^2y}{dx^2}\right)^2$ is

(A) order 1, degree 3

(B) order 2, degree 3

(C) order 1, degree 2

(D) order 2, degree 2

e. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A^2 is

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

f. The lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular to each other if

(A) $m_1 = m_2$

(B) $m_1 = -m_2$

(C) $m_1m_2 = -1$

(D) $m_1m_2 = 1$

g. The center and radius of the circle $(x-3)^2 + (y+4)^2 = 3$ is

(A) $(-3, 4), 3$

(B) $(3, -4), 3$

(C) $(3, -4), \sqrt{3}$

(D) $(-3, -4), \sqrt{3}$

h. The value of $9C_2 + 9C_3$ is equal to

(A) $9C_3$

(B) $10C_2$

(C) $10C_3$

(D) $9C_2$

i. The value of $1 + \cos 4\theta$ is

(A) $2 \cos^2 2\theta$

(B) $2 \cos^2 \theta$

(C) $2 \cos 2\theta$

(D) $2\sqrt{\cos 2\theta}$

j. The maximum value of $4\cos^2\theta + 3\sin^2\theta$ is

(A) 7

(B) 6

(C) 4

(D) 3

Answer any FIVE Questions out of EIGHT Questions.

Each Question carries 16 marks.

Q.2 a. Find the values of a and b such that $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$. (8)

b. Find the n^{th} derivative of $e^x(2x+3)^3$. (8)

Q.3 a. Integrate $\int e^x \frac{1 - \sin x}{1 - \cos x} dx$. (8)

b. Evaluate $\int \frac{x}{(x+2)^2(x-1)} dx$. (8)

- Q.4** a. Discuss the consistency of the following system of equations:
 $2x + 3y + 4z = 11, x + 5y + 7z = 15, 3x + 11y + 13z = 25.$
 If found consistent, then solve them. (8)
- b. Find the Inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$ by using adjoint of the matrix. (8)
- Q.5** a. Solve the differential equation $\left(x \tan \frac{y}{x} - y \sec^2 \frac{y}{x}\right) dx - x \sec^2 \frac{y}{x} dy = 0.$ (8)
- b. Solve the differential equation $x \frac{dy}{dx} + y = x^3 y^6.$ (8)
- Q.6** a. Use the Principle of mathematical induction to show that
 $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1.$ (8)
- b. Find the coefficient of x^7 in the expansion of $\left(\frac{x^2}{2} - \frac{2}{x}\right)^8.$ (8)
- Q.7** a. If $\sin\theta + \sin\phi = \sqrt{3}(\cos\phi - \cos\theta)$, then show that $\sin 3\theta + \sin 3\phi = 0.$ (8)
- b. Solve the equation $\sin\theta + \sin 3\theta + \sin 5\theta = 0.$ (8)
- Q.8** a. Find the equation of the straight line which passes through the intersections of the lines $5x - 6y = 1$ and $3x + 2y + 5 = 0$ and is perpendicular to the line $3x - 5y + 11 = 0.$ (8)
- b. Find the value of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is
 (i) parallel to x-axis (ii) parallel to y-axis (iii) passing through origin. (8)
- Q.9** a. Find the vertex, focus and directrix of the parabola $4x^2 = 9y.$ (8)
- b. Find the equation of the ellipse whose axes are along the coordinate axes, vertices are $(0, \pm 10)$ and eccentricity $e = \frac{4}{5}.$ (8)