ROLL NO. ___________________

Code: DE105/DC105 **Subject: ENGINEERING MATHEMATICS II**

DipIETE – ET/CS (New Scheme)

Time: 3 Hours **June 2019** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The value of $\underset{x\to 0}{Lt}\dfrac{1-\cos x}{3x^2}$ is

(A) 1 **(B)** 0

(C) $\dfrac{1}{6}$ **(D)** $\dfrac{1}{3}$

b. The value of the definite integral $\int_{-a}^{a}|x|dx$ is equal to

(A) a **(B)** a^2

(C) 0 **(D)** $2a$

c. The particular Integral of the differential equation $(D-2)^2 y = e^{2x}$ is

(A) $\dfrac{x^2}{2}e^{2x}$ **(B)** $-\dfrac{x^2}{2}e^{2x}$

(C) $\dfrac{x^2}{2}e^{-2x}$ **(D)** $-\dfrac{x^2}{2}e^{-2x}$

d. The imaginary part of $(\sin x + i\cos x)^5$ is

(A) $-\cos 5x$ **(B)** $-\sin 5x$

(C) $\sin 5x$ **(D)** $\cos 5x$

e. If $\vec{A} = 2i + 2j - k$, $\vec{B} = 6i - 3j + 2k$, then the unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ is

(A) $i + 10j + 18k/5\sqrt{17}$ **(B)** $i - 10j + 18k/5\sqrt{17}$

(C) $-i - 10j - 18k/5\sqrt{17}$ **(D)** $i - 10j - 18k/5\sqrt{17}$

f. Two vectors A and B are perpendicular to each other if and only if

(A) $A \times B = 0$ **(B)** $A \bullet B = 0$

(C) $A \times B \neq 0$ **(D)** $A \bullet B \neq 0$

g. If $x + iy = \sqrt{2} + 3i$ then $x^2 + y$ is equal to

(A) 7 **(B)** 5

(C) 13 **(D)** $\sqrt{2} + 3$

h. $\lim_{n \to \infty} \left(\frac{n}{n-1} \right)^2$ is equal to

(A) 1 **(B)** ∞

(C) 2 **(D)** 0

i. If value of $L\{F(t)\} = f(s)$, then $L\{(\cosh at)F(t)\}$ is equal to

(A) $\frac{1}{2}\left[f(s-a) - f(s+a)\right]$ **(B)** $\frac{1}{2}\left[f(s-a) + f(s+a)\right]$

(C) $-\frac{1}{2}\left[f(s-a) - f(s+a)\right]$ **(D)** $-\frac{1}{2}\left[f(s+a) + f(s-a)\right]$

j. $L^{-1}\left\{ \frac{1}{(s-a)^n} \right\}$ is equal to

(A) $\frac{e^{at}\mathbf{t}^{n-1}}{n!}$ **(B)** $\frac{e^{at}\mathbf{t}^{n}}{(n-1)!}$

(C) $\frac{e^{at}\mathbf{t}^{n-1}}{(n-1)!}$ **(D)** $\frac{e^{at}\mathbf{t}^{n}}{n!}$

Answer any FIVE Questions out of Eight Questions.
Each question carries 16 marks.

Q.2 a. Verify Lagrange's Mean value theorem for the function $f(x) = x^3 - 3x - 1$ in $(-11/7, 13/7)$. **(8)**

b. Evaluate $\lim_{x \to 0} \frac{2\sin x - \sin 2x}{x^3}$. **(8)**

Q.3 a. Evaluate $\int_0^{\frac{\pi}{2}} \cos^9 x dx$. **(8)**

b. Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. **(8)**

Q.4 a. If z is a complex number, then prove that:

(i) $|z|^2 = |\bar{z}|^2 = z\bar{z}$ (ii) $Amp.z + Amp.\bar{z} = 0$. **(8)**

b. If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, Prove that $\frac{x-y}{x+y} = i\tan\frac{\alpha-\beta}{2}$. **(8)**

Q.5 a. If $\vec{A} = i + 2j - 3k$, $\vec{B} = 3i - j + 2k$. Find the angle between $2\vec{A} + \vec{B}$ and $\vec{A} + 2\vec{B}$. **(8)**

b. Evaluate $\vec{A} \bullet (\vec{B} \times \vec{C})$, if $\vec{A} = 2i - 3j$, $\vec{B} = i + j + k$ and $\vec{C} = 3i - k$. **(8)**

Q.6 a. Solve the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\sin^2 2x$. **(8)**

b. Solve the differential equation $\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$. **(8)**

Q.7 a. Discuss the convergence of the series $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} \ldots \infty$ **(8)**

b. Test the convergence of the series $\sum \frac{(n+1)^n x^n}{n^{n+1}}$. **(8)**

Q.8 a. Find the Laplace transform of $[t^2 \sin^2 3t]$. **(8)**

b. Find the Laplace transform of $\left[\frac{1 - \cos 2t}{t^2}\right]$. **(8)**

Q.9 a. Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s^2}{(s^4 - 16)}\right]$. **(8)**

b. Using Laplace transform solve the differential equation $Y'' - 2Y' + Y = e^t$, given that $Y(0) = 2$ and $Y'(0) = -1$. **(8)**