

Time: 3 Hours

June 2019

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following:

(2×10)

a. $y[n] = nx[-n]$ is a

- (A) Non causal system
 (B) Non Linear, time invariant system
 (C) Linear, time variant and dynamic system
 (D) None of these

b. What is the fundamental period of the signal $x(t) = 2\cos(4\pi t) + 3\sin(3\pi t)$?

- (A) 1/2 (B) 2/3
 (C) 3/4 (D) 2

c. If $x(t)$ is the continuous time signal with a period 'T₀' and Fourier coefficients a_n then average power 'P' of $x(t)$ is given by

- (A) $\sum_{n=-\infty}^{\infty} |a_n|^2$ (B) $\frac{1}{T_0} \int_{T_0} |x(t)|^2 dt$
 (C) Both (A) and (B) (D) None of these

d. Continuous Fourier transform of the signal $x(t) = u(-t)$ is

- (A) $\frac{1}{j\omega}$ (B) $-\frac{1}{j\omega}$
 (C) $j\omega$ (D) None of these

e. Consider an LTI system with impulse response $h[n] = \delta[n - n_0]$. The frequency response is

- (A) $e^{-\omega n_0}$ (B) $e^{\omega n_0}$
 (C) 1 (D) None of these

f. A continuous time signal $x(t)$ is obtained at the output of the ideal filter with cut off frequency $\omega_c = 1000\pi$. If impulse train sampling is performed on $x(t)$, which of the following sampling periods would guarantee that $x(t)$ can be recovered from its sampled version using an appropriate low-pass filter?

- (A) $T = 0.5 \times 10^{-3}$ (B) $T = 10^{-4}$
 (C) $T = 2 \times 10^{-3}$ (D) None of these

g. Laplace transform of $x(t) = \frac{\sin t}{t}$ is

- (A) $X(s) = \frac{\pi}{2} - \tan^{-1} s$ (B) $X(s) = \frac{\pi}{2} + \tan^{-1} s$
 (C) $X(s) = -\frac{\pi}{2} - \tan^{-1} s$ (D) None of these

h. Z-transform of $y[n] = n^2 x[n]$ is

- (A) $Y[z] = z \frac{d^2}{dz^2} X[z]$ (B) $Y[z] = -z^2 \frac{d^2}{dz^2} X[z]$
 (C) $Y[z] = z^2 \frac{d^2}{dz^2} X[z]$ (D) None of these

i. Z-transform and ROC of $u(n-1)$ are

- (A) $\frac{z}{z-1}$ and $|z| > 1$ (B) $\frac{1}{z-1}$ and $|z| > 1$
 (C) $\frac{z}{z-1}$ and $|z| < 1$ (D) $\frac{1}{z-1}$ and $|z| < 1$

j. Identify the correct statement regarding probability distribution function ($F_X(x)$)

- (A) $F_X(x)$ is left and right continuous
 (B) $F_X(-\infty) = -1$
 (C) $F_X(\infty) = 0$
 (D) $F_X(x)$ is non decreasing function

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Determine and plot the Even and Odd component of the signal $x(t) = tu(t+2) - tu(t-1)$. (8)

b. Determine the continuous time convolution of $h(t)$ and $x(t)$

$$x(t) = \begin{cases} 3, & 0 \leq t \leq 4 \\ 0, & \text{otherwise} \end{cases} \quad h(t) = \begin{cases} 2, & 0 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

- Q.3** a. If $x_1(t)$ and $x_2(t)$ are two continuous-time signals with common period T_0 and Fourier series coefficients a_n and b_n , respectively, then Fourier series coefficient of convolution integral of $x_1(t)$ and $x_2(t)$ are given by $T_0 a_n b_n$. (8)
- b. Find the Fourier series coefficients of the signal given below and plot the magnitude and phase spectra. (8)
- $$x(t) = 2 \sin(2\pi t - 3) + \sin(6\pi t)$$
- Q.4** a. Find the Fourier transform of the signal $x(t) = \frac{2}{1+t^2}$. (8)
- b. Find Fourier transform of the continuous time signal given below. (8)
- $$x(t) = \frac{t^{n-1}}{(n-1)!} e^{-bt} u(t)$$
- Q.5** a. Find the Fourier transform of the following discrete time signal. (8)
- $$x[n] = a^{|n|}, \quad |a| < 1$$
- b. Explain about the following properties of discrete time Fourier transform. (8)
- (i) First difference of signal $x[n]$ (ii) Time reversal
(iii) Time expansion (iv) Differentiation in frequency
- Q.6** a. Explain about the time domain and frequency domain aspects of non-ideal filters. (8)
- b. Explain about sampling with zero order hold. (8)
- Q.7** a. Find the Laplace transform of the following signal. (8)
- $$x(t) = \frac{e^{4t} - e^{-3t}}{t}$$
- b. Find the inverse Laplace transform of the following (8)
- $$X(s) = \frac{s^2 + s + 1}{s^2 - s + 1}$$
- Q.8** a. Determine the response $y(n), n \geq 0$ of the system described by the second-order difference equation $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ For the input $x[n] = 4^n u[n]$. (8)
- b. Find the inverse z-transform for the following (8)
- $$X(z) = \frac{az^{-1}}{(1-az^{-1})^2}, \quad |z| > |a|$$
- Q.9** a. If X and Y are two random variable and $Z = X + Y$ then show that $\text{VAR}(Z) = \text{VAR}(X) + \text{VAR}(Y) + 2\text{COV}(X, Y)$. (8)
- b. The power spectral density of a WSS white noise process limited in a range $-w \leq f \leq w$ is given as $\frac{N_0}{2}$. Find the average power and autocorrelation function. (8)