ROLL NO. ____________________

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **June 2019** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Which of the following is an analytic function:

(A) $f(z) = \sin z$ **(B)** $f(z) = \bar{z}$
(C) $f(z) = \operatorname{Im}(z)$ **(D)** $f(z) = \operatorname{Re}(iz)$

b. The value of $\int_C \frac{1}{z-1} dz$, C being the circle $|z| = 2$, is

(A) πi **(B)** $-\pi i$
(C) $2\pi i$ **(D)** $-2\pi i$

c. The harmonic conjugate of $e^x \cos y$ is

(A) $e^y \cos x$ **(B)** $-e^x \cos y$
(C) $-e^x \sin y$ **(D)** $e^x \sin y$

d. The solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} = xy$ is

(A) $z = \frac{x^2 y}{3} + x\phi(y) + \varphi(y)$ **(B)** $z = \frac{x^3 y}{6} + x\phi(y) + \varphi(y)$
(C) $z = \frac{x^2 y}{3} + \phi(y) + \varphi(y)$ **(D)** $z = \frac{x^2 y}{3} + \varphi(y)$

e. The partial differential equation formed by eliminating the arbitrary function from $z = f(x^2 - y^2)$ is

(A) $px + qy = 0$ **(B)** $px - qy = 0$
(C) $py + qx = 0$ **(D)** $py - qx = 0$

f. The value of the constants a, b, c so that the vector field $F = (x + 2y + az) I + (bx - 3y - z) J + (4x + cy + 2z) K$ is irratational is:

(A) $a = 4, b = 2, c = -1$ **(B)** $a = -4, b = 2, c = -1$
(C) $a = 4, b = -2, c = -1$ **(D)** $a = -4, b = 2, c = 1$

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g. The acceleration of a particle at any time t is given by A = 12 cos 2*t* *I* – 8 sin 2*t* *J* + 16*t* *K*. If the velocity *V* and displacement *r* are zero at *t* = 0, then *V* is equal to
(A) $V = 6\cos 2t\,I + 4\cos 2t\,J + 8t^2\,K$
(B) $V = 6\sin 2t\,I + 4(\sin 2t - 1)J + 8t^2\,K$
(C) $V = 6\sin 2t\,I + 4(\cos 2t - 1)J + 8t^2\,K$
(D) $V = 6\sin 2t\,I + 4(\sin 2t)\,J + 8t^2\,K$

h. Which of the following is correct, (if the interval of differencing *h* = 1)
(A) $\Delta x^n = nx^{n-1}$ **(B)** $\Delta x^{(n)} = nx^{(n-1)}$
(C) $\Delta^n e^x = e^x$ **(D)** $\Delta \cos x = -\sin x$

i. The Probability that *A* passes a test is 2/3 and the Probability that *B* passes a test is 3/5. The Probability that one of them passes the test is
(A) 2/5 **(B)** 4/15
(C) 7/15 **(D)** 2/15

j. If $f(X) = X + \frac{2}{k}, X = 1, 2, 3, 4, 5$ is the probability distribution of a discrete random variable, then *k* is equal to
(A) 2/7 **(B)** –2/7
(C) 5/7 **(D)** –5/7

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the bilinear transformation which maps the points *z* = 1, *i*, –1 onto the points ω = *i*, 0, –*i*. **(8)**

b. Determine the analytic function $f(z) = u + iv, if\ u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$. **(8)**

Q.3 a. Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices –1, 1, 1+*i*, –1+*i*. **(8)**

b. Find the residue of $f(z) = \frac{z^3}{(z-1)^3(z-2)(z-3)}$ at its poles and hence evaluate $\oint_C f(z)dz$, where *C* is the circle |z|=2.5. **(8)**

Q.4 a. Prove that:
(i) $\nabla \times \nabla f = 0$
(ii) $\nabla \times (\nabla \times F) = \nabla(\nabla \bullet F) - \nabla^2 F$. **(8)**

b. Find the value of β if the vector $(\beta x^2 y + yz)I + (xy^2 - xz^2)J + (2xyz - 2x^2 y^2)K$ has zero divergence. **(8)**

Q.5 a. Verify Green's theorem for $\int_C \left[\left(3x-8y^2\right)dx+\left(4y-6xy\right)dy\right]$ where C is the boundary of the region bounded by $x = 2$, $y = 0$ and $x + y = 1$. **(8)**

b. Evaluate $\int_S F \bullet ds$ where $F = 4\,x\,I - 2y^2\,K + z^2\,K$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(8)**

Q.6 a. If $f\,(1.15) = 1.0723$, $f\,(1.20) = 1.0954$, $f\,(1.25) = 1.1180$ and $f\,(1.30) = 1.1401$, find $f\,(1.28)$. **(8)**

b. Given that

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log x$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6484

Evaluate $\int_4^{5.2} \log x\,dx$ (i) Trapezoidal rule and (ii) Simpson's $1/3$ rule. **(8)**

Q.7 a. Solve the partial differential equation $x^2\,(y - z)\,p + y^2\,(z - x)\,q = z^2\,(x - y)$. **(8)**

b. Find the complete integral of the partial differential equation $(p^2 + q^2)\,y = q\,z$ by Charpit's method. **(8)**

Q.8 a. A pair of dice is tossed twice. Find the probability of scoring 7 points:
(i) once (ii) at lease once
(iii) twice **(8)**

b. State and prove Baye's theorem. **(8)**

Q.9 a. A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k,
(ii) Evaluate $P\,(X < 6)$, $P\,(X \geq 6)$, $P\,(3 < X \leq 6)$
(iii) Find the minimum value of x, so $P\,(X \leq x) > \frac{1}{2}$. **(8)**

b. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, 1 defective and 2 defective blades in a consignment of 10,000 packets. **(8)**