ROLL NO. ___________________

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours | **June 2019** | **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 to Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTIONS FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If $x = r\cos\theta$, $y = r\sin\theta$, the $\frac{\partial(x, y)}{\partial(r, \theta)}$ is equal to

(A) r^2 **(B)** r

(C) $\frac{1}{r}$ **(D)** None of these

b. Two functions $u(x, y)$ and $v(x, y)$ are functionally dependent if

(A) Their product is real **(B)** Their Jacobian is not zero

(C) Their Jacobian is zero **(D)** None of these

c. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is

(A) 1 **(B)** 2

(C) 3 **(D)** None of these

d. If $|A| = 0$, then the matrix A has

(A) At least one eigen value zero **(B)** None of the eigen value zero

(C) All the eigen value unity **(D)** None of these

e. By applying elementary transformation to a matrix, its rank

(A) Increases **(B)** Decreases

(C) does not change **(D)** None of these

f. The rate of convergence of Regula-falsi method is

(A) 1 **(B)** 2

(C) 1.6 **(D)** None of these

g. The particular integral of $(D^2 + 4)y = \cos 2x$ is

(A) $-\frac{x}{4} \sin 2x$ **(B)** $\frac{x}{4} \sin 2x$

(C) $\frac{x}{4} \cos 2x$ **(D)** None of these

h. The value of the integral $\int_0^{\infty} e^{-x^2} dx$ is

(A) $\frac{2}{\sqrt{\pi}}$ **(B)** $\frac{\sqrt{\pi}}{2}$

(C) $\frac{\pi}{2}$ **(D)** None of these

i. The solution of the differential equation

$(y^2 + 1)x\,dx + (x^2 + 1)y\,dy = 0$, is

(A) $(x^2 + 1)(y^2 + 1) = c$ **(B)** $(x^2 + 1) = c(y^2 + 1)$

(C) $(x^2 + y^2 + xy) = c$ **(D)** None of these

j. The value of $J_{1/2}(x)$ is

(A) $\sqrt{\frac{2}{\pi x}} \sin x$ **(B)** $\sqrt{\frac{2}{\pi x}} \cos x$

(C) $\sqrt{\frac{\pi x}{2}} \sin x$ **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q. 2. **a.** If $u(x, y) = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. **(8)**

b. Find the minimum distance from the point (1, 2, 0) to the cone $z^2 = x^2 + y^2$. **(8)**

Q. 3. **a.** Find by double integration, the area lying between the curves
$y = 4x - x^2$ and $y = x$. **(8)**

b. Evaluate $\int_0^1 \int_{e^x}^{e} \frac{dy\,dx}{\log y}$ by changing the order of integration. **(8)**

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Q. 4 **a.** Determine the values of λ and μ such that the system $2x - 5y + 2z = 8$, $2x + 4y + 6z = 5$, $x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution (iii) infinite number of solutions. **(8)**

b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **(8)**

Q. 5 **a.** Prove that Newton-Raphson method has second order convergence. **(8)**

b. Find a real root of the equation $\cos x = x\, e^x$ by using the method of false position correct to four decimal places. **(8)**

Q. 6 **a.** Solve $\frac{d^2y}{dx^2} + 2\,\frac{dy}{dx} + 10\,y + 37\sin 3x = 0$ and find the value of y when $x = \frac{\pi}{2}$ being given that $y = 3$, $\frac{dy}{dx} = 0$ when x = 0. **(8)**

b. Solve: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2\sin(\log x)$. **(8)**

Q. 7. **a.** Solve in series the differential equation: $\frac{d^2y}{dx^2} + x^2y = 0$. **(8)**

b. Prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ **(8)**

Q.8 **a.** State and prove the orthogonality of Bessel's functions. **(8)**

b. Express $f(x) = x^4 + 3x^3 + 2x^2 + 6x + 3$ in terms of Legendre polynomials. **(8)**

Q. 9. (For Current Scheme students i.e. AE51/AC51/AT51)

a. Prove that $x\,J'_n = x\,J_{n-1} - n\,J_n$. **(8)**

b. Solve the differential equation: $\frac{dy}{dx} + y\sec x = \tan x$. **(8)**

Q. 9. (For New Scheme students i.e. AE101/AC101/AT101)

a. Obtain the half-range sine series for the function $f(x) = x^2$ in the interval $0 < x < 3$. **(8)**

b. Use Z-transform to solve the equation $y_{n+2} + 4\,y_{n+1} + 3\,y_n = 3^n$ with $y_0 = 0,\ y_1 = 1$. **(8)**