ROLL NO. ________________

Code: AC65/AC116 Subject: DISCRETE STRUCTURES

# AMIETE – CS (Current & New Scheme)

Time: 3 Hours **June 2019** Max. Marks: 100

***PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.***

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

---

**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a. $A-(A-B)$ is

**(A)** Equal to $A$ **(B)** Equal to $A\cup B$

**(C)** Equal to $B$ **(D)** Equal to $A\cap B$

b. The truth value of $(P\to Q)\to R$ is $F$ if $P,Q$ and $R$ have truth values

**(A)** $F,T,T$ **(B)** $T,T,T$

**(C)** $T,T,F$ **(D)** $F,T,F$

c. If $P(x,y)$ is $3y+2<x$ and $R$ is the universe of discourse, then the truth value of $\exists x\forall y P(x,y)$ and $\forall x\exists y P(x,y)$ are

**(A)** T and T **(B)** T and F

**(C)** F and T **(D)** F and F

d. The value of the Fibonacci number $F_{10}$ is

**(A)** 10 **(B)** 55

**(C)** 34 **(D)** 89

e. Let $A=\{1,2,3\}$. The number of binary relations on $A$ is equal to

**(A)** 2 **(B)** 8

**(C)** 256 **(D)** 512

f. The relation $R$ defined in $Z$ by $mRn$ if $mn\geq 0$ is

**(A)** Reflexive, symmetric and transitive

**(B)** Not reflexive, symmetric and transitive

**(C)** Reflexive, symmetric but not transitive

**(D)** Reflexive, transitive but not symmetric

g. If $f$ is function from $Z$ to $Z$ defined by $f(n)=n+2$, then $f^{-3}(10)$ is

**(A)** 7 **(B)** 6

**(C)** 5 **(D)** 4

h. If $H$ is the only coset of $H$ in $G$, then one of the following statements is not true:

**(A)** $G = H$ **(B)** $H$ is normal in $G$

**(C)** $[G:H] = 1$ **(D)** $G$ is abelian

i. $R = \{0, \pm4, \pm8, \ldots\}$ under usual addition and multiplication is

**(A)** Not a ring **(B)** A commutative ring

**(C)** Acommutative ring with identity **(D)** A field

j. The number of relations from $A$ to $B$ with $|A| = m$ and $|B| = n$ is

**(A)** $mn$ **(B)** $2^n$

**(C)** $2^m$ **(D)** $2^{mn}$

---

**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

---

**Q.2** a. Using Venn diagram prove that, for any three sets $A, B, C$, $A\Delta(B\Delta C) = (A\Delta B)\Delta C$. **(8)**

b. Three students $X, Y, Z$ write an examination. Their chances of passing are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Find the probability that (i) all of them pass, (ii) at least one of them passes, and (iii) at least two of them pass. **(8)**

.

**Q.3** a. Express the following propositions in terms of only NAND and only NOR connectives.
$(i)\neg p \quad (ii) p\wedge q \quad (iii) p\vee q \quad (iv) p \to q$ **(8)**

b. Simplify the switching network represented by
$u \equiv [p\vee q\vee r]\wedge[p\vee t\vee \neg q]\wedge[p\vee \neg t\vee r]$ **(8)**

**Q.4** a. Consider the following open statements with the set of all real numbers as the universe.
$p(x): x \geq 0, q(x): x^2 \geq 0, r(x): x^2 - 3x - 4 = 0, s(x): x^2 - 3 > 0$
Determine the truthness or falsity of the following statements.
$(i)\exists x, p(x)\wedge q(x) \quad (ii)\forall x, p(x) \to q(x) (iii)\forall x, q(x) \to s(x) (iv)\forall x, r(x)\vee s(x)$ **(8)**

b. Let $m$ and $n$ be integers. Prove that $n^2 = m^2$ if and only if $m = n$ or $m = -n$. **(8)**

**Q.5** a. Prove, by mathematical induction, that
$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ for all integers $n \geq 1$. **(6)**

b. If $A = \{1,2,3\}, B = \{2,3,4\}, C = \{3,4,5\}$ verify that
$A \times (B \cup C) = (A \times B) \cup (A \times C)$ and $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **(5)**

**c**. If $R = \{x, y / x > y\}$ is a relation on the set $A = \{1,2,3,4\}$. Write down the matrix and the digraph of $R$. **(5)**

**Q.6** a. Consider the poset $(\wp(S), \subseteq)$, where $S = \{1,2,3,4\}$. Draw the Hasse diagram of the poset and find all maximal and all minimal elements. Determine the LUB and GLB of $B = \{\{1\}\{1,2\}\{1,3\}\{1,2,3\}\}$. **(8)**

b. Verify that $R = \{(1,1),(2,2),(3,3),(4,4),(1,2,(2,1)\}$ is an equivalence relation on the set $A = \{1,2,3,4\}$. Find the corresponding partition of $A$. **(8)**

**Q.7** a. Let $f, g, h$ be function from $R$ to $R$ defined by $f(x) = x^2, g(x) = x + 5$ and $h(x) = \sqrt{x^2 + 2}$. Verify that $(h \circ g) \circ f = h \circ (g \circ f)$. **(8)**

b. Let $A = B = R$ and $D = \{x, y/y = x^2\}$. Prove that $\pi_A(D)$ is one-to-one and onto whereas $\pi_B(D)$ is neither one-to-one nor onto**.** **(8)**

**Q.8** a. If $\circ$ is an operation on $Z$ defined by $x \circ y = x + y + 1$. Prove that $(Z, \circ)$ is an abelian group. **(8)**

b. In the group $(U_{16}, *)$, find all $x$ other than 1 and 15 for which $x = x^{-1}$. **(8)**

**Q.9** a. Prove that the set $Z$ with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1, x \odot y = x + y - xy$ is a commutative ring with unity. **(8)**

b. An encoding function $E: Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix $G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$. Determine all the code words. Find the associated parity-check matrix $H$. Use $H$ to decode the received words: 11101,11011 **(8)**