

DiplETE – ET/CS (Current & New Scheme)

Time: 3 Hours

Jun-2018

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following:a (2×10)

a. If $y = x^2 - \cos x - \frac{1}{x^2}$, then $\frac{dy}{dx}$ is,

(A) $x - \cos x + \frac{2}{x^3}$

(B) $2x - \sin x + \frac{2}{x^3}$

(C) $2x + \sin x - \frac{2}{x^3}$

(D) $2x + \sin x + \frac{2}{x^3}$

b. Let A and B be two matrices, then the relation $(AB)^n = A^n B^n$, if

(A) $AB = BA$

(B) $AB \neq BA$

(C) $A = B$

(D) $A^{-1} = B$

c. The differential equation of the family of curves $y = e^x(A \cos x + B \sin x)$, where A and B are arbitrary constants, is

(A) $\frac{dy}{dx} + Ax + By = 0$

(B) $\frac{dy}{dx} - Ax - By = 0$

(C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

(D) None of these

d. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{9}{41}$ where $0 < A < \frac{\pi}{2}$, $0 < B < \frac{\pi}{2}$ then the value of $\cos(A-B)$ is

(A) $\frac{106}{107}$

(B) $-\frac{156}{205}$

(C) $\frac{156}{205}$

(D) None of these

e. . If $\Delta = \begin{vmatrix} 2x-1 & x+7 & x+4 \\ x & 6 & 2 \\ x-1 & x+1 & 3 \end{vmatrix}$, then the value of Δ in the respect of x is equal to,

- (A) 2, 3, 5
 (B) 1, 3, 4
 (C) 1, 2, 3
 (D) 1, 2, 4

f. The order (O) and degree (D) of differential equation of $y \frac{d^2x}{d^2y} = y^2 + 1$ is

- (A) O = 2, D = 1
 (B) O = 0, D = 1
 (C) O = 1, D = 1
 (D) O = 1, D = 2

g. $\int \sin^2 x \cdot \cos^2 x \, dx$ is equal to

- (A) $\frac{1}{8} \left(x - \frac{\sin 4x}{4} \right) + c$
 (B) $\frac{1}{8} \left(x + \frac{\sin 4x}{4} \right) + c$
 (C) $\frac{1}{8} \left(x - \frac{\cos 4x}{4} \right) + c$
 (D) $\frac{1}{8} \left(x + \frac{\cos 4x}{4} \right) + c$

h. The solution of the differential equation $\frac{dy}{dx} = xy^2 - xy$ is equal to

- (A) $\log \frac{y-1}{y} = \frac{x^2}{2} + c$
 (B) $\log \frac{y-1}{y} = x + 2 + c$
 (C) $\log \frac{y+1}{y} = \frac{x^2}{2} + c$
 (D) $\log \frac{y+1}{y} = x - 2 + c$

i. . If $y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$ then $\frac{dy}{dx}$ is

- (A) $-\frac{1}{2}$
 (B) $\frac{1}{2}$
 (C) 1
 (D) -1

j. The equation of the line which makes intercepts -4 and 5 on the axis is:

- (A) $4x + 5y + 20 = 0$
 (B) $4x - 5y + 20 = 0$
 (C) $5x - 4y + 20 = 0$
 (D) $5x + 4y + 20 = 0$

**Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.**

Q.2 a. Differentiate the following functions: **(8)**

(i) $x^n e^x \log_e x$ (ii) $\cos ec^{-1}\left(\frac{x^2+1}{x^2-1}\right) + \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

b. Find the equation of the tangent to the curve $x^2 + 2y = 8$ which is perpendicular to the line $x - 2y + 1 = 0$. **(8)**

Q.3 a. Evaluate $\int x \cos^3 x dx$ **(8)**

b. Evaluate $\int \frac{1}{\sqrt{x(1-2x)}} dx$ **(8)**

Q.4 a. Apply Cramer's rule to solve the following system of linear equations:

$$3x - 2y + 4z = 5$$

$$x + y + 3z = 2$$

$$-x + 2y - z = 1$$

(8)

b. Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$$
 (8)

Q.5 a. Solve the differential equation $(x + y) dy + (x - y) dx = 0$ given that $y = 1$ when $x = 1$ **(8)**

b. Solve the differential equation $\frac{dy}{dx} + \frac{1 + \cos 2y}{1 - \cos 2x} = 0$. **(8)**

Q.6 a. Find the term independent of x in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ **(8)**

b. Find three numbers in A.P. whose sum is 21 and their product is 315. **(8)**

Q.7 a. Prove that, $\cos 2A \cdot \cos 2B + \sin^2(A - B) - \sin^2(A + B) = \cos (2A + 2B)$ (8)

b. If $A + B + C = \pi$, prove that $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$ (8)

Q.8 a. Find the equation of the lines through the origin and making an angle of 60° with the line $x + y\sqrt{3} + 3\sqrt{3} = 0$ (8)

b. Find the area of the triangle formed by the lines $y = x$, $y = 2x$ and $y = 3x + 4$ (8)

Q.9 a. Find the equation to the circle, which passes through the point $(-2, 4)$ and through the points in which the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is cut by the line $3x + 2y - 5 = 0$. (8)

b. Find the equation of the parabola with focus $(3, -4)$ and the directrix $6x - 7y + 5 = 0$ (8)