ROLL NO. ________________

**Code: AE51/AC51/AT51/AE101/AC101/AT101**
**Subject: ENGINEERING MATHEMATICS - I**

## AMIETE – ET/CS/IT (Current & New Scheme)

**Time: 3 Hours** **June 2018** **Max. Marks: 100**

***PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.***

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 TO Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTIONS FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

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**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a. If $x = r\cos\theta, y = r\sin\theta$ then $\frac{\partial(r,\theta)}{\partial(x,y)} =$

**(A)** r **(B)** -r
**(C)** 1/r **(D)** -1/r

b. $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z)\,dy\,dx\,dz =$

**(A)** 0 **(B)** $1$
**(C)** $-2$ **(D)** $x$

c. One of the eigen value of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$ is

**(A)** 0 **(B)** 11
**(C)** 24 **(D)** 4

d. Newton's –Rapson formula will converge only if------------ in the interval is considered.

**(A)** $|\phi(x)| < 1$ **(B)** $|f(x)f''(x)| < |f'(x)|^2$
**(C)** $|\phi(x)| > 1$ **(D)** $|f(x)f''(x)| > |f'(x)|^2$

e. The particular integral of $(D-2)^2 y = 8e^{2x}$ is

**(A)** $e^{2x}$ **(B)** $\frac{x^2}{2} e^{2x}$
**(C)** $4x^2 e^{2x}$ **(D)** 0

f. $\Gamma\left(\frac{1}{2}\right) =$

**(A)** 1.772 **(B)** 3.14
**(C)** 2.71 **(D)** 0.5

g. The complete solution is $y = c_1(y)_{m_1} + c_2(y)_{m_2}$ (where $m_1, m_2$ are the roots)

**(A)** when roots of the indicial equations are distinct and do not differ by an integer
**(B)** when roots of the indicial equations are distinct and differ by an integer
**(C)** when roots of the indicial equations are equal
**(D)** when roots of the indicial equations are unity

h. If $u = f_1(x,y,z), v = f_2(x,y,z), w = f_3(x,y,z)$, the necessary and sufficient condition that $u, v, and\ w$ are functionally related is $\frac{\partial(u,v,w)}{\partial(x,y,z)} =$

**(A)** 0 **(B)** 1
**(C)** -1 **(D)** 3

i. The rank of coefficient matrix and augmented matrix are same but it is less than the number of unknowns, then the system is

**(A)** inconsistent
**(B)** consistent and there is a unique solution
**(C)** consistent and there are infinite number of solutions
**(D)** there is no solution

j. Solving ordinary differential equation by modified Euler's method is equivalent to solve by Runge-Kutta method of

**(A)** first order **(B)** second order
**(C)** third order **(D)** fourth order

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**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

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**Q.2** a. If $u = \sin^{-1}\frac{x+2y+3z}{x^8+y^8+z^8}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. **(4)**

b. Expand $e^x \log(1+y)$ in powers of $x$ and $y$ upto terms of third degree. **(5)**

c. A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. **(7)**

**Q.3** a. Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\, dydx$ and hence evaluate the same. **(8)**

b. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. **(8)**

**Q.4** a. Test for consistency and solve,
5x + 3y + 7z = 4; 3x + 26y + 2z = 9; 7x + 2y + 10z = 5 **(8)**

b. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(8)**

**Q.5** a. Find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using power method. Take $[1 \quad 0 \quad 0]^T$ as initial eigen vector. **(8)**

b. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at x = 0.2, 0.4 **(8)**

**Q.6** a. Solve, by the method of variation of parameters, $y'' - 2y' + y = e^x \log x$. **(8)**

b. Solve the simultaneous equations $\frac{dx}{dt} + 2y + \sin t = 0, \frac{dy}{dt} - 2x - \cos t = 0$ given that $x = 0$ and $y = 1$ when $t = 0$. **(8)**

**Q.7** a. Establish the relation between Beta function and Gamma function. And also express the integral $\int_0^1 \frac{dx}{\sqrt{(1-x^4)}}$ in terms of gamma function. **(8)**

b. Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0$ about the origin. **(8)**

**Q.8** a. Compute the values of $J_{\frac{1}{2}}(x)$ and $J_{-\frac{1}{2}}(x)$. **(8)**

b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(8)**

**Q.9** **(For Current Scheme students i.e. AE51/AC51/AT51)**

a. Solve $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$. **(8)**

b. Solve $(1 + 2xy \cos x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0$. **(8)**

**Q.9** **(For New Scheme students i.e. AE101/AC101/AT101)**

a. If $f(x) = |\cos x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$. **(8)**

b. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$, using Z-transforms. **(8)**