ROLL NO. ___________________

Code: AC65/AC116 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current & New Scheme)

Time: 3 Hours **June 2018** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Which of the following is not equal to null set
(A) {x|x is an integer and $x^2+4=6$}
(B) {x|x is an integer and 3x+5=9}
(C) {x|x is a real number and $x^3 = -1$}
(D) {x|x is a real number and x = x+1}

b. Which of the following represents a partition of the set of natural numbers?
(A) [{x:x < 5}, {x:x >5}] **(B)** [{x:x > 5}, {0}, {1, 2, 3, 4, 5}]
(C) [{x:x^2 < 11}, {x:x^2 >11}] **(D)** [{1, 3, 5}, {2, 4, 7, 9}, {0, 6, 8}]

c. Which one of the following is not necessarily a property of group?
(A) Associativity
(B) Existence of identity
(C) Existence of inverse for every statement
(D) Commutativity

d. Which of the following statement is the negation of the statement,
"2 is even and -3 is negative"?
(A) 2 is even and -3 is not negative
(B) 2 is odd and -3 is not negative
(C) 2 is odd or -3 is not negative
(D) 2 is even or -3 is not negative

e. Let I^+ be the set of positive integers and R be the relation on I^+ by xRy iff $2x \le y-1$. Then which ordered pair belongs to R?
(A) (9, 3) **(B)** (3, 9)
(C) (3, 2) **(D)** (2, 2)

f. The inverse of $\sim p \to q$ is
(A) $q \to \sim p$ **(B)** $\sim p \to \sim q$
(C) $p \to \sim q$ **(D)** $\sim q \to \sim p$

g. If A and B are Mutually exclusive events, then
(A) $A \cap B \neq \phi$ **(B)** $A \cap B = \phi$
(C) $A \cup B = \phi$ **(D)** $A \cup B \neq \phi$

h. If A and B are finite sets with |A| = |B| and $f : A \rightarrow B$ then which of the following is not equivalent
(A) f is one-to-one **(B)** f is onto
(C) f is a bijection **(D)** f is a partial function

i. Which of the following is not true about cyclic groups:
(A) Every cyclic group is abelian
(B) Every abelian group is cyclic
(C) If g is a generator of a cyclic group, then g^{-1} is also a generator of this group
(D) Klein-4-group is not cyclic

j. For all $x, y, z \in Z_2^m$ which is not true
(A) $d(x, y) = d(y, x)$ **(B)** $d(x, y) \neq d(y, x)$
(C) $d(x, y) \geq 0$ **(D)** $d(x, y) = 0$ if and only if $x=y$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Check the validity of the following argument:- **(8)**
"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the labour market is not perfect."

b. If A, B, C are finite sets, prove that **(8)**
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Q.3 a. In each case below, say whether the statement is a tautology, a contradiction or neither. **(8)**
(i) $p \vee \sim (p \rightarrow p)$ (ii) $(p \rightarrow \sim p) \vee (\sim p \rightarrow p)$
(iii) $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$ (iv) $(p \wedge q) \vee (\sim p \vee \sim q)$

b. Express the statement $(\sim (p \vee q)) \vee ((\sim p) \wedge q)$ in simplest possible form. **(8)**

Q.4 a. What do you mean by recurrence relation? Solve the following recurrence relation: **(8)**
$$a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$$

b. There are two restaurants next to each other. One has a sign that says, "Good food is not cheap" and the other has a sign that says, "Cheap food is not good". Are the signs saying the same thing? **(4)**

c. Show that $\neg \forall x [P(x) \rightarrow Q(x)]$ and $\exists x [P(x) \wedge \neg Q(x)]$ are logically equivalent. **(4)**

Q.5 a. Suppose U is a universal set and A, B1, B2, ……, Bn ⊆ U. Prove that
A∩(B1 ∪ B2 ∪ …… ∪Bn) = (A∩B1) ∪ (A∩B2) ∪ …….. (A∩Bn) **(8)**

b. Prove by induction **(8)**
1.1! + 2.2! + …….+n.n! = (n+1)! – 1

Q.6 a. Define the following (i) Reflexive (ii) Symmetric (iii) Transitive properties of Relation with an example. **(3+3+2)**

b. Prove that if *a* and *b* are elements in a bounded distributive lattice and if *a* has a complement a', then **(2x4)**
(i) $a \vee (a' \wedge b) = a \vee b$
(ii) $a \wedge (a' \vee b) = a \wedge b$

Q.7 a. Consider the functions *f* and *g* defined by **(8)**
$f(x) = x^3$
$g(x) = x^2 + 1, \ \forall x \in R$
Find $f \circ g$, $g \circ f$, f^2, g^2

b. Consider the function f: N → N, where N is the set of natural numbers, defined by $f(n) = n^2 + n + 1$. Show that the function *f* is one-one but not onto. **(8)**

Q.8 a. Let S = {0, 1, 2, 3, 4, 5, 6, 7} and ⊗ denotes multiplication modulo 8, that is x ⊗ y = (x.y) mod 8. **(8)**
(i) Prove that ({0, 1}, ⊗) is not a group.
(ii) Write three distinct groups (G, ⊗) where G ⊂ S and G has two elements.

b. How many generators are there of the cyclic group of order 8? **(8)**

Q.9 a. Write short note on Generator Matrix **(8)**

b. Give steps and an example to generate a parity check matrix. **(8)**