ROLL NO. ___________________

Code: DE55/DC55 Subject: ENGINEERING MATHEMATICS - II

DipIETE – ET/CS (Current Scheme)

Time: 3 Hours **JUNE 2016** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Laplace transform of $\frac{\cos at - \cos bt}{t}$ is :

(A) $\log\frac{s^2+b^2}{s^2+a^2}$ **(B)** $\frac{1}{2}\log\frac{s^2+a^2}{s^2+b^2}$

(C) $\frac{1}{2}\log\frac{s^2+b^2}{s^2+a^2}$ **(D)** $\log\frac{s+b}{s+a}$

b. The value of the limit $Lim_{x\to\infty}\frac{e^x}{x^3} =$

(A) 0 **(B)** 1
(C) 2 **(D)** ∞

c. The constant term in the Fourier series for the function $f(x) = x^2$ in the interval $(-\pi, \pi)$ is :

(A) $\frac{\pi^2}{3}$ **(B)** $\frac{\pi^3}{3}$

(C) $\frac{2\pi^2}{3}$ **(D)** $\frac{2\pi^3}{3}$

d. The expression $e^{5+\frac{\pi i}{2}}$ in the form of $A + iB$ is :

(A) $2 - ie^2$ **(B)** ie^5
(C) $1 - 3e^{-3}$ **(D)** $1 - 4i$

e. A particular integral of the differential equation $\left(D^2+4\right)y = x$ is :

(A) xe^{-2x} **(B)**) $x\cos 2x$

(C) $x\sin 2x$ **(D)** $\frac{x}{4}$

f. If $a = 2i - j + k$, $b = i - 3j - 5k$ and $c = 3i - 4j - 4k$ then $a \cdot (b \times c)$ is :

(A) $2i - 3j - k$ **(B)** $-3j - k$
(C) $-k$ **(D)** 0

g. If $L\{f(t)\} = \bar{f}(s)$ then $L\{e^{at} f(t)\}$ is :

(A) $\bar{f}(s+a)$ **(B)** $\bar{f}(s-a)$
(C) $e^{as}\bar{f}(s)$ **(D)** $e^{-as}\bar{f}(s)$

h. If f is a function such that $Lim_{x\to 2} \dfrac{f(x) - f(2)}{x-2} = 0$, which of the following must be true?

(A) The limit of $f(x)$ as x approaches 2 does not exist
(B) f is not defined at $x = 2$
(C) The derivative of f at $x = 2$ is zero
(D) f is continuous at $x = 0$

i. The solution of $\left(D^4 + 8D^2 + 16\right)y = 0$ is given by :

(A) $c_1e^{2x} + c_2e^{-2x} + c_3e^{x} + c_4e^{-x}$
(B) $(c_1 + c_2x)e^{2x} + (c_3 + c_4x)e^{-2x}$
(C) $(c_1 + c_2x)\cosh 2x + (c_3 + c_4x)\sinh 2x$
(D) $(c_1 + c_2x)\sin 2x + (c_3 + c_4x)\cos 2x$

j. If $a = 2i - 3j - k$ and $b = i + 4j - 2k$ then $(a+b)\times(a-b)$ is :

(A) $-20i - 6j - 22k$ **(B)** $-22i + 6j - 2k$
(C) $20i + 16j - 22k$ **(D)** $-2i - 6j - 2k$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. State Mean-Value theorem. Also verify it for the function $f(x) = 2x^2 - 7x + 10$, $a = 2, b = 5$. **(8)**

b. Expand $\tan^{-1} x$ in powers of x by Maclaurin's theorem. **(8)**

Q.3 a. Evaluate $\int_0^1 x^4(1-x^2)^{\frac{5}{2}}\,dx$ **(8)**

b. Find the area of the segment cutoff from the parabola $y^2 = 2x$ by the straight line $y = 4x - 1$. **(8)**

Q.4 a. Prove that the points $x+iy$ and $\frac{1}{-x+iy}$ lies on a straight line through the origin. **(8)**

b. If n is an integer then show that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$. **(8)**

Q.5 a. A, B, C, D are the points $i-k$, $-i+2j$, $2i-3k$, $3i-2j-k$ respectively. Show that the projection of AB on CD is equal to that of CD on AB. Also find the cosine of their inclination. **(8)**

b. Find the cosine of the angle between the direction of the vectors $a = 6i+2j+3k$ and $b = 3i-2k$. Also find a unit vector perpendicular to both a and b. **(8)**

Q.6 a. Solve the differential equation $\left(D^2-2D+5\right)y = e^{2x}\sin x$ **(8)**

b. The deflection of a strut of length b with one end $(x=0)$ built-in and the other supported and subjected to end thrust P, satisfies the equation $\frac{d^2y}{dx^2} + a^2 y = \frac{a^2R}{P}(b-x)$. Prove that the deflection curve is $y = \frac{R}{P}\left[\frac{\sin ax}{a} - b\cos ax + b - x\right]$ where $ab = \tan ab$. **(8)**

Q.7 a. Find the Fourier series for the function $f(x) = \begin{cases} \pi x & for \quad 0 \le x < 1 \\ \pi(2-x) & for \quad 1 \le x < 2 \end{cases}$ **(8)**

b. Develop $\sin\frac{\pi x}{l}$ in a half-range cosine series in the range $0 \le x \le l$. **(8)**

Q.8 a. Find Laplace transform of $\frac{1-\cos 2t}{t}$. **(8)**

b. Solve the following differential equation using Laplace transform :
$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$, given $y(0)=0, y'(0)=1$ **(8)**

Q.9 a. Apply convolution theorem to evaluate $L^{-1}\left\{ \frac{s}{(s^2+a^2)^2} \right\}$ **(8)**

b. Find inverse Laplace transform of $\frac{(s+2)}{(s^2+4s+5)^2}$. **(8)**