ROLL NO. ___________________

Code: CT22 Subject: DISCRETE MATHEMATICAL STRUCTURES

ALCCS

Time: 3 Hours **JUNE 2016** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

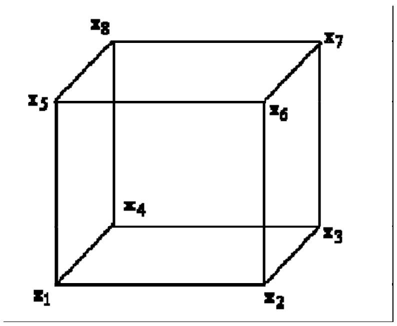
- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

Q.1 a. Consider the set A = {2, 7, 14, 28, 56, 84} and the relation a ≤ b if and only if a divides b. Give the Hasse diagram for the poset (A, ≤).

b. By using pigeonhole principle, show that if any five numbers from 1 to 8 are chosen, then two of them will add up to 9.

c. Define tautology and contradiction with an example of each.

d. Define Hamilton path. Determine if the following graph has a Hamilton circuit.



e. Let T be a binary tree with n vertices. Determine the maximum number of leaf nodes in the tree.

f. Determine the values of the following prefix expressions. (↑ is exponentiation.)
+, -, ↑ , 3, 2, ↑ , 2, 3, /, 6, –, 4, 2

g. State the relation between Regular Expression, Transition Diagram and Finite State Machines. **(7×4)**

Q.2 a. Let the universe of discourse be the set of integers. Then consider the following predicates:

$P(x): x^2 \geq 0$

$Q(x): x^2 - 5x + 6 = 0$

$R(x,y): x^2 = y$

Now find all the well formed formula from the following expressions. Find the truth value of each.

(i) $\forall x\,[P(x) \wedge Q(x)]$

(ii) $\forall x\ \forall y\,[R(P(x), y) \rightarrow P(x)]$

(iii) $\forall x\ P(x) \wedge \exists y\, Q(y)$

(iv) $\forall y\, \forall x\,[P(x) \rightarrow R(x, y)]$

(v) $\forall y\, \exists x\, R(y, x) \vee [\,\forall z Q(z)]$ **(10)**

Q.2 b. If R is a relation N×N defined by (a,b) R (c,d) iff a + d = b + c, show that R is an equivalence relation. **(8)**

Q.3 a. If A and B are two subsets of a Universal set, then prove that $A - B = A \cap \overline{B}$. **(6)**

b. Define symmetric, asymmetric and antisymmetric relations giving example in each category. **(6)**

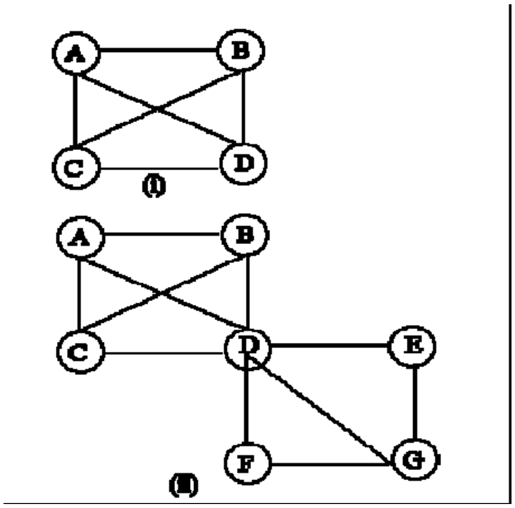
c. Show that if $R1$ and $R2$ are equivalence relations on A, then $R1 \cap R2$ is an equivalence relation. **(6)**

Q.4 a. Let A = $\{1, 2, 4, 8, 16\}$ and relation R_1 be partial order of divisibility on A. Let $A' = \{0, 1, 2, 3, 4\}$ and R_2 be the relation "less than or equal to" on integers. Show that (A, R_1) and (A', R_2) are isomorphic posets. **(9)**

b. If $[L, \wedge, \vee]$ is a complemented and distributive lattice, then the complement a' of any element $a \in L$ is unique. **(9)**

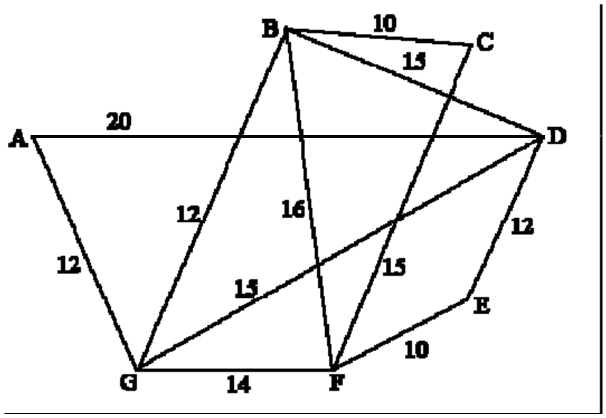
Q.5 a. Prove that a simple graph is connected if and only if it has a spanning tree. **(8)**

b. Define Euler Circuit and Euler Path. Which of the following graphs have an Euler circuit and Euler path. **(6)**



c. State and explain Kuratoski's theorem. **(4)**

Q.6 a. Use Kruskal's algorithm to find a minimal spanning tree for the following graph. **(8)**



b. Let A = {1, 2, 3, 4} and, R = {(1, 1), (1, 4), (2, 1), (2, 2), (3, 3), (4, 4)}.Use Warshall's algorithm to find the transitive closure of R. **(10)**

Q.7 a. Construct a grammar G to generate the language over an alphabet $\{0, 1\}$ with an equal number of 0's and 1's for some integer $n \geq 0$. **(9)**

b. Test whether 101101, 11111 are accepted by a finite state machine M given as follows: $M = [Q = \{q_0, q_1, q_2, q_3\}, \sum = \{0,1\}, \delta, \{q_0\}]$
where the transition function δ is **(9)**

	Inputs	
States	0	1
→ q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2