ROLL NO. _______________

Code: AE56/AC56/AT56/AE107/AC107/AT107
Subject: ENGINEERING MATHEMATICS – II

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **JUNE 2016** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2x10)**

a. The value of the integral $\int_C \frac{e^{-z}}{z+1}\,dz$, where C is $|z| = \frac{1}{2}$ is

(A) $2\pi i$ **(B)** $3\pi i$
(C) πi **(D)** 0

b. The singularity of $f(z) = \sin\frac{1}{1-z}$ at $z = 1$ is

(A) Pole **(B)** Zeros
(C) Isolated essential singularity **(D)** Non-isolated essential singularity

c. The function $f(z) = \frac{1}{z^n}$ is

(A) Analytic everywhere **(B)** Singularity at $z = 0$
(C) Singularity at $n = 0$ **(D)** None of these

d. If $\vec{A} = t\,i - 3j + 2t\,k$, $\vec{B} = i - 2j + 2k$, $\vec{D} = 3i + tj - k$ then the value of $\int_1^2 \vec{A}.(\vec{B} \times \vec{D})\,dt$ is equal to

(A) 1 **(B)** 2
(C) 0 **(D)** None of these

e. The magnitude of the gradient of the function $f = xyz^3$ at (1, 0, 2) is

(A) 2 **(B)** 8
(C) 4 **(D)** None of these

f. The value of $\Delta^6(ax+b)(cx^2+d)(ex^3+f)$ is

(A) ace 6! **(B)** abcdef 6!
(C) bdf 6! **(D)** 0

g. The missing term in the following table is

x:	2	3	4	5	6
f(x):	45.0	49.2	54.1	…….	67.4

(A) 60.05 **(B)** 59.64
(C) 64.02 **(D)** None of these

h. The solution of $yzp - xzq = xy$ is

(A) $f(x^2 + y^2,\ x^2 + z^2) = 0$ **(B)** $f(x^2 + y^2,\ x^2 - z^2) = 0$

(C) $f\left(\frac{x}{y},\ \frac{y}{z}\right) = 0$ **(D)** None of these

i. What is the probability of throwing a total of 11 with two dice if the digit on the first die is 5 ?

(A) $1/4$ **(B)** $1/6$

(C) $1/5$ **(D)** $1/2$

j. The probability density function f(x) of a continuous random variable *x* is defined as $f(x) = \begin{cases} \frac{A}{x^2}, & 5 \le x \le 10 \\ 0, & otherwise \end{cases}$, The value of A is

(A) $150/3$ **(B)** $100/3$

(C) $200/3$ **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 **a**. Define Analytic function and then prove that an analytic function with constant modulus is constant**.** **(8)**

b. (i) Find an analytic function whose imaginary part is $e^{-x}(x\cos y + y\sin y)$. **(4)**

(ii) Find the Bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. **(4)**

Q.3 **a**. State and prove Cauchy's theorem**.** **(8)**

b. Evaluate $\int_C \frac{e^{2}+z}{z^3-z}\,dz,\ where\ C\ is\ |z| = \frac{\pi}{2}$. **(8)**

Q.4 a. A particle moves on the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction of $i - 3j + 2k$. **(8)**

b. Show that $\nabla(\vec{A}.\vec{B}) = (\vec{A}.\vec{\nabla})\vec{B} + (\vec{B}.\vec{\nabla})\vec{A} + \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A})$. **(8)**

Q.5 **a**. Use Green's theorem in a plane to evaluate the integral $\oint_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary in the xy-plane of the area enclosed by the $x - axis$ and the semi-circle $x^2 + y^2 = 1$ in the upper half of *xy*-plane. **(8)**

b. Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = (x - 3y)i + (y - 2x)j$ and C is closed curve in the xy-plane given by $x = 2\cos t,\ y = 3\sin t$ from $t = 0\ to\ t = 2\pi$. **(8)**

Q.6 **a.** From the following table

x:	1	2	3	4	5	6	7	8
f(x):	1	8	27	64	125	216	343	512

Estimate $f(7.5)$. **(8)**

b. From the following table

x:	-1	0	2	3	7	10
f(x):	-11	1	1	1	141	561

find interpolating polynomial $f(x)$ and hence evaluate $f'(6), f''(5), f'''(6).$ **(8)**

Q.7 a. Solve the equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$ **(8)**

b. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0,$ given that, when $x = 0,\ z = e^y$ and $\frac{\partial z}{\partial x} = 1.$ **(8)**

Q.8 a. A bag *X* contains 2 white and 3 red balls and a bag *Y* contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag *Y*. **(8)**

b. A problem in mechanics is given to three students *A*, *B* and *C* whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved? **(8)**

Q.9 a. If mean and variance of a binomial distribution are 4 and 2 respectively, find the probability of (i) exactly 2 success (ii) less than 2 success (iii) at least 2 success. **(8)**

b. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poission distribution to calculate the approximate number of packets containing no defective blade, one defective blade and two defective blades respectively in a consignment of 10,000 packets. **(8)**