ROLL NO. _______________

Code: AE51/AC51/AT51/AE101/AC101/AT101 Subject: Engineering Mathematics-I

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **JUNE 2016** Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 TO Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTION FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $u = \ell og \frac{x^2}{y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(A) x **(B)** y
(C) u **(D)** 1

b. The value of the double integral $\int_0^a \int_{2\sqrt{ay}}^{2a} xy\,dx\,dy$ is

(A) $\frac{a^4}{4}$ **(B)** $\frac{a^4}{3}$
(C) $\frac{a^4}{2}$ **(D)** a^4

c. If the product of two eigen values of $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is -4, then the third eigen value is

(A) 2 **(B)** –2
(C) 3 **(D)** –3

d. The Newton-Raphson iterative formula to find $\sqrt{N}$ is

(A) $x_{n+1}=\frac{1}{2}\left(x_n+\frac{1}{Nx_n}\right)$ **(B)** $x_{n+1}=\frac{1}{2}\left(x_n-\frac{1}{Nx_n}\right)$

(C) $x_{n+1}=\frac{1}{2}\left(x_n+\frac{N}{x_n}\right)$ **(D)** $x_{n+1}=\frac{1}{2}\left(x_n-\frac{N}{x_n}\right)$

e. The solution of $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=0$ is

(A) $y = C_1\cos x + C_2\sin x$
(B) $y = C_1e^{x} + C_2e^{-x}$
(C) $y = (C_1+ C_2x)\cos x + (C_3 + C_4x)\sin x$
(D) $y = (C_1+ C_2x + C_3x^2)\sin x + C_4\sin x$

f. Particular integral of $\frac{d^3y}{dx^3}-\frac{dy}{dx}=e^x+e^{-x}$ is

(A) $\frac{1}{2}\left(e^x+e^{-x}\right)$ **(B)** $\frac{x}{2}\left(e^x+e^{-x}\right)$

(C) $\frac{x^2}{2}\left(e^x+e^{-x}\right)$ **(D)** $\frac{x^2}{2}\left(e^x-e^{-x}\right)$

g. $J^1_{\frac{1}{2}}(x).J_{-\frac{1}{2}}(x)-J^1_{-\frac{1}{2}}(x).J_{\frac{1}{2}}(x)$ is equal to

(A) $\frac{2}{\pi x}$ **(B)** $-\frac{2}{\pi x}$

(C) $\frac{\pi x}{2}$ **(D)** $-\frac{\pi x}{2}$

h. Half range sine series for f(x) = 1 in the range $0<x<\pi$ is

(A) $\frac{4}{\pi}\left(\sin x+\frac{\sin 2x}{2^2}+\frac{\sin 3x}{3^2}+\ldots\ldots\ldots\ldots\right)$

(B) $\frac{4}{\pi}\left(\sin x+\frac{\sin 2x}{2}+\frac{\sin 3x}{3}+\ldots\ldots\ldots\ldots\right)$

(C) $\frac{4}{\pi}\left(\sin x+\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}+\ldots\ldots\ldots\ldots\right)$

(D) $\frac{4}{\pi}\left(\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\ldots\ldots\ldots\ldots\right)$

i. Z-transform of $n\,a^n$ is

(A) $\frac{az}{z-a}$ **(B)** $\frac{az}{z+a}$

(C) $\frac{az}{(z-a)^2}$ **(D)** $\frac{az}{(z+a)^2}$

j. Expansion of $e^x \ell og(1+y)$ in powers of x and y upto first degree terms is

(A) x **(B)** y

(C) x + y **(D)** x - y

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If Z is a homogenous function of degree n in x and y, show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$. **(8)**

b. If $u = x^2 - 2y^2$, $v = 2x^2 - y^2$, where $x = r\cos\theta, y = r\sin\theta$, show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$. **(8)**

Q.3 a. Evaluate the integral by changing the order of integration, **(8)**

$$\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy.$$

b. Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **(8)**

Q.4 a. For what values of k the equations x+y+z = 1, 2x+y+4z = k, 4x +y +10z = k^2 have a solution and solve them in each case. **(8)**

b. Find the eigen values and eigen vectors of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. **(8)**

Q.5 a. Use Regula falsi method to compute the real root, correct to three decimal places, of the equation $xe^x = 2$. **(8)**

b. Solve $\frac{dy}{dx} = y^2 + x$, y(0)=1, using Taylor's series method and compute y(0, 1). **(8)**

Q.6 a. Solve the differential equation $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x \cos 2x$. **(8)**

b. Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\,\ell og\,x}{x^2}$. **(8)**

Q.7 a. Solve in series the equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$. **(8)**

b. Solve that $\int_{-1}^{1} xP_n(n)P_{n-1}(n) = \frac{2n}{4n^2 - 1}$. **(8)**

Q.8 a. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. **(8)**

b. Show that $\int_{-1}^{1} x^2P_{n-1}P_{n+1}dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$. **(8)**

Q.9 **(For Current Scheme students i.e. AE51/AC51/AT51)**

a. Solve $y \log y \, dx + (x - \log y)dy = 0$. **(8)**

b. A body at 80°C cools down to 60°C in 20 min, the temperature of air being 40°C. What will the temp of the body be after 40 min from original time? **(8)**

Q.9 **(For New Scheme students i.e. AE101/AC101/AT101)**

a. Obtain a Fourier series to represent x–x^2 from x=-π to x=π. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$. **(8)**

b. Obtain the constant term and coefficients of first sine and cosine terms in the Fourier expansion of y as given in the following table: **(2+3+3)**

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20