Code: AC65/AC116 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current & New Schemes)

Time: 3 Hours **JUNE 2016** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If S and T are two sets, then ($S \oplus T$) is:

(A) S + T **(B)** $S + T – (S \cap T)$
(C) $(S \cup T) – (S \cap T)$ **(D)** None of these

b. The valid conclusion drawn from the hypothesis $q \rightarrow p$ and ~p is:

(A) p **(B)** ~q
(C) ~p **(D)** q

c. Which of the following represents a partition of the set of natural numbers?

(A) [{x: x < 5}, {x: x > 5}] **(B)** [{1, 3, 5}, {2, 4, 7, 9}, {0, 6, 8}]
(C) [{x: $x^2 < 11$}, {x: $x^2 > 11$}] **(D)** [{x: x > 5}, {0}, {1, 2, 3, 4, 5}]

d. Every cyclic group is :

(A) Abelain group **(B)** Distributive group
(C) Quotient group **(D)** None of these

e. In the Hasse diagram of D_{24} where D_{24} represents the set of divisors of 24 the least and greatest elements are:

(A) (0, 24) **(B)** (2, 24)
(C) (1, 24) **(D)** (24, 24)

f. The contrapositive of $p \rightarrow q$ is the:

(A) Inverse of $q \rightarrow p$ **(B)** Inverse of the converse of $p \rightarrow q$
(C) Converse of $\sim p \rightarrow \sim q$ **(D)** All of these

g. $\sim \exists x\, P(x)$ is equivalent to:

(A) $\forall x\, P(\sim x)$ **(B)** $\forall x \sim P(x)$
(C) $\exists \sim x\, P(x)$ **(D)** $\exists x \sim P(x)$

h. $\overline{(\overline{A}\ U\ B)}$ is equivalent to :

(A) $(\overline{A}\ U\ \overline{B})$ **(B)** $(A \cap \overline{B})$
(C) $\overline{(A\ U\ \overline{B})}$ **(D)** $\overline{(A \cap B)}$

i. If an element has two compliments (if exists) in a bounded lattice then the lattice is not:

(A) Distributive **(B)** Commutative
(C) Poset **(D)** None of these

j. A relation R defined on a set X is antisymmetric if :
(A) xRy => yRx **(B)** xRy and yRx => x = y
(C) xRy and yRx => x ≠ y **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Prove that $P(A \cap B) = P(A) \cap P(B)$ where P(A) is the power set of set A. **(8)**

b. Each user on a computer system has a password which is six to eight characters long, where each character is an alphabet or a digit. Each password must contain at least one digit. How many possible passwords are there? **(8)**

Q.3 a. In each case below, say whether the statement is a tautology, a contradiction or neither. **(8)**

(i) $p \vee \sim (p \to p)$

(ii) $(p \to \sim p) \vee (\sim p \to p)$

(iii) $(p \to \sim p) \wedge (\sim p \to p)$

(iv) $(p \wedge q) \vee (\sim p \vee \sim q)$

b. Show that the following prepositions are equivalent. **(8)**

$p \to (q \to r) \equiv p \to (\sim q \vee r) \equiv (p \wedge q) \to r$

Q.4 a. Define universal and existential quantifiers with example. Determine the truth value for each of the following statements. Assume x, y are elements of set of integers. **(8)**

(i) $\forall x \exists y \quad x + y \text{ is even}$

(ii) $\exists x\ \forall y \quad x + y \text{ is even}$

b. Give the tautological form of modus ponens, modus tollens and hypothetical syllogism and prove the validity of any two. **(8)**

Q.5 a. Compute A(2, 1) when A : N*N → N, where N is the set of natural numbers, is defined by: **(8)**

A(0, y) = y + 1
A(x + 1, 0) = A(x, 1)
A(x + 1, y + 1) = A(x + 1, y)

b. By mathematical induction, prove that $n! \geq 2^{n-1}$ for all integers $n \geq 1$. **(8)**

Q.6 a. Let A be a set and P(A) be the power set of A. Relation R defined on P(A) as XRY if $X \subseteq Y$, where X, Y $\in$ P(A). Prove that R is a partial order relation. **(8)**

b. Draw the Hasse diagram of (D_{100}, /) where D_{100} represents the set of divisors of 100 and relation / (divides) is a partial order relation. Also find the greatest lower bound (GLB) and least upper bound (LUB) of the following sub-sets (if exists): (i) {10, 20} (ii) {4, 20, 25} **(8)**

Q.7 a. In each case, say whether the function is one-to-one and whether it is onto. **(8)**
(i) $f : Z \times Z \rightarrow Z \times Z$, defined by $f(a, b) = (a + b, a - b)$
(ii) $f : R \times R \rightarrow R \times R$, defined by $f(a, b) = (a + b, a - b)$

b. If $f: Q \rightarrow Q$ such that $f(x) = 2x$ and $g: Q \rightarrow Q$ such that $g(x) = x + 2$ are two functions then verify that $(gof)^{-1} = f^{-1} \text{ o } g^{-1}$. **(8)**

Q.8 a. Define monoid. Find whether the set Q^+ of non-zero positive rational numbers form a monoid for the composition * defined by $a*b = a / b$, for all $a, b \in Q^+$ **(8)**

b. Let $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $\otimes$ denotes multiplication modulo 8, that is $x \otimes y = (x.y) \bmod 8$. **(8)**
(i) Prove that $(\{0, 1\}, \otimes)$ is not a group.
(ii) Write three distinct groups $(G, \otimes)$ where $G \subset S$ and G has two elements.

Q.9 Write short notes on any TWO of the followings: **(8×2)**
(i) Hamming distance
(ii) Parity-check matrix
(iii) The ring Z_n