Subject: ENGINEERING MATHEMATICS - I

Time: 3 Hours **JUNE 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. $\lim_{x\to 0} \dfrac{\tan x - \sin x}{\sin^3 x}$ is :

(A) $-\dfrac{1}{2}$ **(B)** $\dfrac{1}{2}$

(C) 1 **(D)** 0

b. If $y = \log(\sec x + \tan x)$ then $\dfrac{dy}{dx}$ is

(A) $-\sec x$ **(B)** $\tan x$

(C) $-\tan x$ **(D)** $\sec x$

c. $\int \dfrac{\sin 4x}{\sin x}$ is

(A) $2\left(\dfrac{\sin 3x}{3} + \sin x\right) + C$ **(B)** $\dfrac{\sin 3x}{3} + \sin x + C$

(C) $\dfrac{\sin 3x}{3} - \sin x + C$ **(D)** $2\left(\dfrac{\sin 3x}{3} - \sin x\right) + C$

d. $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, then $A^2 - 4A + 3I$ is

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & -1 \\ 0 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

e. If $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$, then value of Δ is

(A) $(b-c)(c-a)(a-b)$ **(B)** 0

(C) a+b+c **(D)** $-a-b-c$

f. The order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = 5\frac{d^2y}{dx^2}$ is

(A) Order = 2, Degree = 2 **(B)** Order = 1, Degree = 2

(C) Order = 2, Degree = 1 **(D)** Order = 1, Degree = 1

g. The term independent of x in the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ is

(A) $\frac{19}{18}$ **(B)** $\frac{5}{18}$

(C) $\frac{11}{18}$ **(D)** $\frac{7}{18}$

h. The value of $\operatorname{cosec} 2\theta + \cot 2\theta$ is

(A) $\cot 3\theta$ **(B)** $\tan\theta$

(C) $\cot\theta$ **(D)** $\tan 2\theta$

i. The equation of the straight line which makes an angle of 45° with x – axis and cuts of an intercept 3 on y – axis above origin

(A) $x-y+3=0$ **(B)** $x+y+3=0$

(C) $2x+y-3=0$ **(D)** $x-2y+3=0$

j. The centroid of the triangle with vertices $(2,7)$, $(3,4)$ and $(-6,4)$ is

(A) $\left(\frac{2}{3},5\right)$ **(B)** $\left(\frac{1}{3},5\right)$

(C) $\left(-\frac{2}{3},5\right)$ **(D)** $\left(-\frac{1}{3},5\right)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that, $\cos 20^\circ \cos 30^\circ \cos 40^\circ \cos 80^\circ = \frac{\sqrt{3}}{16}$ **(8)**

b. In a triangle ABC, if $\sin 2A + \sin 2B = \sin 2C$, prove that either $A = 90^\circ$ or $B = 90^\circ$. **(8)**

Q.3 a. Find the middle terms in the expansion of $\left(3x - \frac{x^3}{6}\right)^7$. **(8)**

b. The fourth, seventh and the last term of a G.P. are 10, 80 and 2560 respectively. Find the first term and the number of terms in G.P. **(8)**

Q.4 a. Show that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$. **(8)**

b. Using matrix method, solve the following system of equation and also check whether the system is consistent or inconsistent $x + y + z = 6;\ 2x - y + z = 3;$ $x - 2y + 3z = 6$ **(8)**

Q.5 a. A line passes through (3, 4) and the sum of its intercepts on the axis is 14, find the equation of the line. **(8)**

b. If p is the length of the perpendicular from the origins to the line $\frac{x}{a} + \frac{y}{b} = 1$, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$. **(8)**

Q.6 a. Find the equation of the circle which passes through the points (1,1) & (2,2) and whose radius is 1. **(8)**

b. Find the equation of the ellipse having axis along coordinate axis, passing through (4, 3) & (-1, 4). **(8)**

Q.7 a. If $y = \sin\left(m\sin^{-1}x\right)$, prove that

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$$ **(8)**

b. Show that $\frac{\log x}{x}$ has a maximum value at x = e. **(8)**

Q.8 a. Evaluate $\int \operatorname{cosec}^3 x\,dx$. **(8)**

b. Evaluate $\int_0^{\pi/2} \frac{1}{3\sin^2 x + 4\cos^2 x}\,dx$ **(8)**

Q.9 a. Solve the initial value problem:

$$\sin x\cos y\,dx + \cos x\sin y\,dy = 0, y(0) = \frac{\pi}{4}$$ **(8)**

b. Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$. **(8)**