Code: DE23/DC23 **Subject: MATHEMATICS - II**
Time: 3 Hours **JUNE 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The modulus of the complex number $\cos\theta + i\sin\theta$ is

(A) -1 **(B)** 1
(C) 2 **(D)** 0

b. The principal argument of 2 is

(A) 0 **(B)** 1
(C) -1 **(D)** i

c. The scalar product of the vectors $\hat{i}+3\hat{j}-8k$ and $-3\hat{i}-5\hat{j}+4\hat{k}$ is

(A) 60 **(B)** -55
(C) -50 **(D)** 45

d. The adjoint of the square matrix $\begin{bmatrix} 7 & 4 \\ 3 & -6 \end{bmatrix}$ is

(A) $\begin{bmatrix} -6 & -4 \\ -3 & 7 \end{bmatrix}$ **(B)** $\begin{bmatrix} -6 & 7 \\ -3 & -4 \end{bmatrix}$
(C) $\begin{bmatrix} -6 & -3 \\ -4 & 7 \end{bmatrix}$ **(D)** $\begin{bmatrix} -3 & -6 \\ 7 & -4 \end{bmatrix}$

e. The product of $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 2 & 4 \end{bmatrix}$ is

(A) $\begin{bmatrix} 9 & 6 \\ -6 & 2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & 9 \\ 3 & 6 \end{bmatrix}$
(C) $\begin{bmatrix} -6 & 9 \\ 2 & 6 \end{bmatrix}$ **(D)** $\begin{bmatrix} 6 & 9 \\ -2 & 2 \end{bmatrix}$

f. The value of $\begin{vmatrix} 1 & 5 & 27 \\ 1 & 5 & 40 \\ 1 & 5 & 18 \end{vmatrix}$ is

(A) 125 **(B)** 200
(C) 0 **(D)** 225

g. The value $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix}$ is

(A) 0 **(B)** $\frac{1}{2}$
(C) 1 **(D)** -1

h. The solution of the differential equation $(D^2 + 4)y = 0$ is

(A) $y = A\cos 2x + B\sin 2x$ **(B)** $y = e^x(A\cos^2 2x + B\sin^2 2x)$
(C) $y = (A_1 + A_2)\cos^2 x + (A_3 + A_4)\sin^2 x$ **(D)** $y = (A_1 + A_2 x)\cos 2x + (A_3 + A_4)\sin 2x$

i. The value $L^{-1}\left[\frac{s}{s^2 + a^2}\right]$ is

(A) $\frac{1}{a}\cos at$ **(B)** $\cosh at$

(C) $\cos at$ **(D)** $\frac{1}{a}\cosh at$

j. The function $f(x) = e^x$ has the Fourier expansion $e^x = \sum_1^\infty b_n \sin nx$ in the interval $(0, \pi)$. Then $\sum_1^\infty (b_n)^2$ converges to

(A) $\frac{1}{\pi}(e^\pi - 1)$ **(B)** $\frac{1}{\pi}(e^\pi + 1)$
(C) $\frac{1}{\pi}(e^{2\pi} - 1)$ **(D)** $\frac{1}{\pi}(e^{2\pi} + 1)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $\frac{a + ib}{c + id} = x + iy$, prove that $x^2 + y^2 = \frac{a^2 + b^2}{c^2 + d^2}$ **(8)**

b. If $Z_1=2+7i$ and $Z_2=1-5i$, then verify that (i) $|Z_1 Z_2| = |Z_1||Z_2|$

(ii) $\left|\dfrac{Z_1}{Z_2}\right| = \dfrac{|Z_1|}{|Z_2|}$ **(8)**

Q.3 a. If $2\cos\theta = x + \dfrac{1}{x}$

$2\cos\phi = y + \dfrac{1}{y}$

Show that one of the value of

$\dfrac{x^m}{y^n} + \dfrac{y^n}{x^m}$ is **(8)**

$2\cos(m\theta - n\phi)$

b. Prove using vectors:
If the diagonals of a parallelogram are equal, then it is a rectangle **(8)**

Q.4 a. Show that $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2$ **(8)**

b. Forces P=2i+5j+6k and Q= –i–2j–k act on a particle. Determine the work done when the particle is displaced from a point A with position vector 4i-3j-2k to a point B with position vector 6i+j–3k. **(8)**

Q.5 a. Show that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **(8)**

b. Use Cramer's Rule to solve the equations
$x+y+z = -1$
$x+2y+3z = -4$
$x+3y+4z = -6$ **(8)**

Q.6 a. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2-4A-5I=0$ **(8)**

b. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. Hence find A^{-1} **(8)**

Q.7 a. Find the Fourier series for $f(x) = \pi + x$ in $(-\pi, \pi)$. **(8)**

b. Find the Fourier series for $f(x) = x^2$ in $(-\pi, \pi)$. Hence derive value of $\sum \frac{(-1)^{n-1}}{n^2}$. **(8)**

Q.8 a. Find the Laplace transform of $\sin\left(\sqrt{t}\right)$. **(8)**

b. Find the inverse Laplace transform of $\frac{s^2+1}{s^3+3s^2+2s}$. **(8)**

Q.9 a. Solve the differential equation, $\left(D^2 - 2D + 5\right)y = e^{2x}\sin x$; where $D = \frac{d}{dx}$. **(8)**

b. Use Laplace transform method to solve

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$$

with

$x(0) = 2$

$x'(0) = -1$ **(8)**