Subject: ENGINEERING MATHEMATICS - II

Time: 3 Hours **JUNE 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. $curl(x\hat{i} + y\hat{j} + z\hat{k})$ is equal to

(A) 0 **(B)** 1
(C) -1 **(D)** None of these

b. Residue of $\frac{\cos z}{z}$ at z = 0 is

(A) 1 **(B)** -1
(C) 2 **(D)** 0

c. When a vibrating string has an initial velocity, its initial conditions are

(A) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$ **(B)** $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v$
(C) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \infty$ **(D)** None of these

d. If $\phi = 3x^2 y - y^3 z^2$, $grad\phi$ at (1,-2,-1) is equal to

(A) -(12i+9j+16k) **(B)** (12i+5j+8k)
(C) -(12i-5j+8k) **(D)** -(12i+5j-8k)

e. Image of $|z+1| = 1$ under the mapping w = 1/z is (where w = u + iv)

(A) 2v+1=0 **(B)** 2v-1=0
(C) 2u+1=0 **(D)** 2u-1=0

f. The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2} = \sin(xy)$ is

(A) $z = -x^2 \sin(xy) + yf(x) + g(x)$ **(B)** $z = -x^2 \sin(xy) - xf(x) + g(x)$
(C) $z = -y^2 \sin(xy) + yf(x) + g(x)$ **(D)** $z = x^2 \sin(xy) + yf(x) + g(x)$

g. In a Poisson distribution if 2P(x = 1) = P(x=2), then the variance is

(A) 4 **(B)** 2
(C) 3 **(D)** 1

h. If variant of the random variable X is 2, then the variant of (2X+3) is

(A) 6 **(B)** -8
(C) 8 **(D)** $2\sqrt{2}$

i. The value of $\Delta^n\left[e^x\right]$, where Δ is a forward operator

(A) $(e+1)^n e^x$ **(B)** $(e-1)^n e^x$
(C) $(e+1)^n e^{-x}$ **(D)** None of these

j. The value of $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's 1/3rd rule (taking n = ¼) is equal to

(A) -0.7845 **(B)** 0.7854
(C) 0.8745 **(D)** 0

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the function $f(z)=\overline{z}$ is continuous at the point z = 0, but not differentiable at z = 0. **(8)**

b. Find the image of the region $|z-i|<2$ under the mapping $w=\frac{1+i}{z+i}$. **(8)**

Q.3 a. Evaluate the integral

$$\int_c (x+y^2-ixy)dz, \text{ where } C: z=z(t)=\begin{cases} t-2i, & 1\le t\le 2 \\ 2-i(4-t), & 2\le t\le 3 \end{cases}$$ **(8)**

b. Show that the function $f(z)=Ln\left[z/z-1\right]$ is analytic in the region $|z|>1$, obtain the Laurent series expansion about z = 0 valid in the region. **(8)**

Q.4 a. If $\nabla.\overline{E}=0$, $\nabla.\overline{H}=0$, $\nabla\times\overline{E}=-\frac{\partial\overline{H}}{\partial t}$, $\nabla\times\overline{H}=\frac{\partial\overline{E}}{\partial t}$, show that vector E and H satisfy the wave equation $\nabla^2 u=\frac{\partial^2 u}{\partial t^2}$ **(8)**

b. Find the values of constants λ and μ so that the surfaces $\lambda x^2-\mu yz=(\lambda+2)x$, $4x^2y+z^3=4$ intersect orthogonally at the point (1,-1,2). **(8)**

Q.5 a. The cylinder $y^2 + z^2 = 9$ intersect the sphere $x^2 + y^2 + z^2 = 25$ find the surface area of the portion of the sphere cut by the cylinder above the yz plane and within the cylinder. **(8)**

b. Use the Divergence theorem to evaluate $\iint_S (\overline{v}.\overline{n})dA, where\, \overline{v} = x^2 z\hat{i} + y\hat{j} - xz^2\hat{k}$, and S is the boundary of the region bounded by the paraboloid z = $x^2 + y^2$ and the plane z = 4y. **(8)**

Q.6 a. Solve by method of separation of variables $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ **(8)**

b. Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ **(8)**

Q.7 a. The following are data from the steam table:

Temp $^\circ$C (t)	140	150	160	170	180
Pressure kgf/cm^2 (P)	3.685	4.854	6.302	8.076	10.225

Using Newton's formula, find the pressure of steam for temperature 142^0 and 175^0. **(8)**

b. A curve is drawn to pass through the following points:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, x-axis and lines x = 1, x = 4. Also find the volume of solid generated by revolving this area using Simpson's 3/8 rule. **(8)**

Q.8 a. A problem in mechanics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved? **(8)**

b. There are three bags: first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white & one red. Find the probability that the balls so drawn came from the second bag. **(8)**

Q.9 a. X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} x^3, & 0 \le x \le 1 \\ (2-x)^3, & 1 \le x \le 2 \end{cases}$$

find the standard deviation and also the mean deviation about the mean. **(8)**

b. A car hire firm has two cars which it hires out day by day. The number of demand for a car on each day is distributed as a Poisson Distribution with mean 1.5. Calculate the proportion of days on which car is not used and the proportion of days on which some demand is refused. (given that $e^{-1.5} = 0.2231$) **(8)**