ROLL NO. ________________

Code: CT22 **Subject: DISCRETE MATHEMATICAL STRUCTURES**

ALCCS

Time: 3 Hours **December - 2017** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

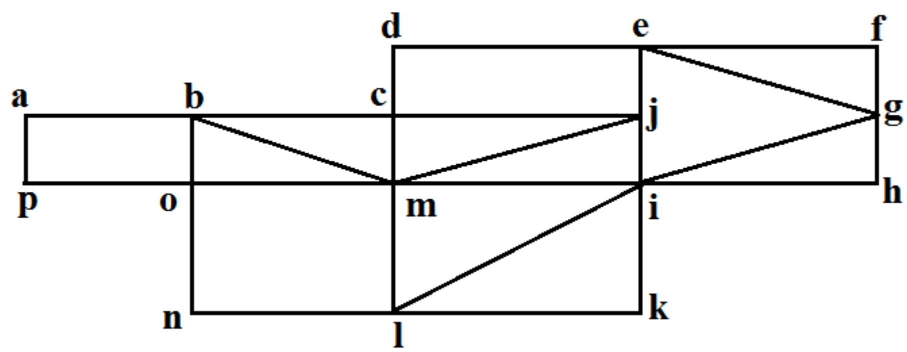
Q.1

a. Prove that in any group of six people, at least three must be mutual friends or at least three must be mutual strangers.

b. Construct the truth table of $(P \to Q) \vee (Q \to P)$

c. Draw the Hasse Diagram of the relation $\subseteq$ on P(A) where $A = \{a, b, c\}, P(A)$ is the power set of A.

d. Does the following graph contains Hamiltonian circuit and Hamiltonian path.



e. Prove that there exist one and only one path between into vertices in a tree.

f. Define
   (i) Distributive lattice.
   (ii) Complimented lattice.

g. Draw the state diagram representing the non-deterministic finite automation (NFA). M is given below
$M = \{(q_0, q_1, q_2, q_3), (0,1), \delta, q_0, (q_3)\}$ **(7×4)**

δ	0	1
q_0	q_0, q_1	q_0, q_2
q_1	q_3	-
q_2	-	q_3
q_3	q_3	q_3

Q.2 a. A survey of 500 television watchers produced the following information: 285 watch football games; 195 watch hockey games; 115 watch basket ball games; 45 watch football and basketball games; 70 watch football and hockey games; 50 watch hockey and basket ball games; 50 do not watch any of the three games
(i) How many people in the survey watch all the three games?
(ii) How many people watch exactly one of the three games? **(9)**

b. If R is the rotation on the set of positive integers such that (a, b) $\in R$, iff $(a^2 + b)$ is even. Prove that R is an equivalence rotation. **(9)**

Q.3 a. Prove that $P \to Q \Leftrightarrow \neg Q \to \neg P$ **(6)**

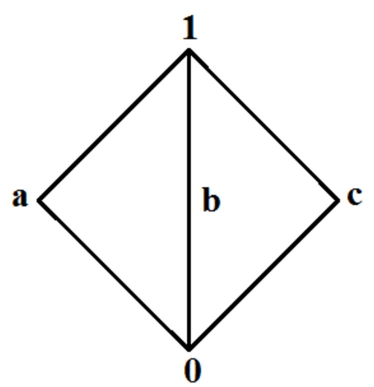
b. Obtain the P D N F of $P \wedge (P \to Q)$ **(6)**

c. Let $(L, \vee \wedge)$ be a Lattice and $a, b, c \in L$. Show that if $b \le a, c \le a$ then $b \vee c \le a$ **(6)**

Q.4 a. In any Boolean Algebra, show that $a \le b \Rightarrow a + bc = b(a + c)$ **(6)**

b. In any Boolean Algebra, show that $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$ **(6)**

c. Prove that the Lattice is not distributive **(6)**

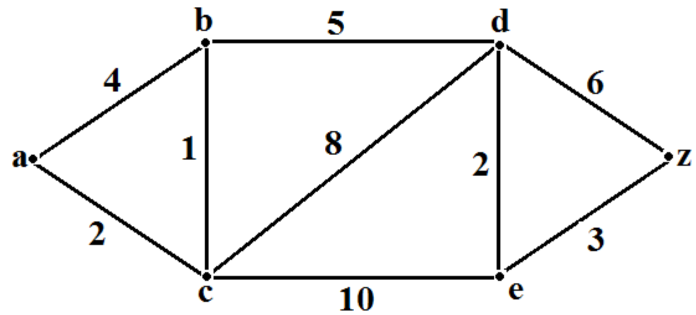


Q.5 a. Show that the maximum number of edges in a simple connected graph with n vertices and k-components is $\frac{1}{2}(n-k)(n-k+1)$ **(9)**

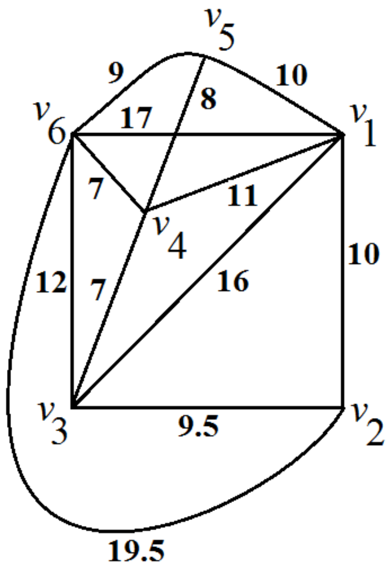
b. Prove that the number of vertices of odd degree in a graph is always even. **(5)**

c. Define work, path and circuit with example. **(4)**

Q.6 a. Use Dijkstra's algorithm to find the shortest path from a to z in the graph given below: **(9)**

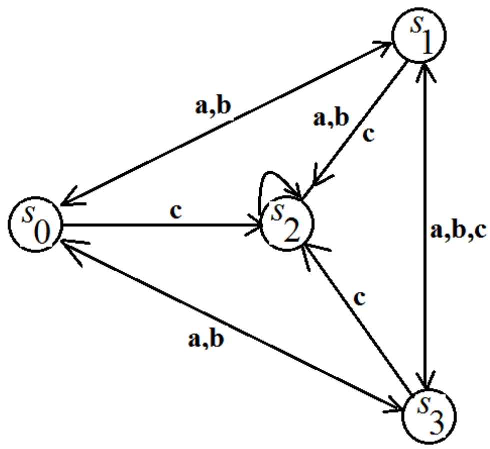


b. Use Prim's Algorithm to find the minimal spanning tree of the following graph: **(9)**



Q.7 a. Explain the different types of grammers. **(6)**

b. Construct the state transition table of the finite state machine whose digraph is given below: **(6)**



c. Convert the given NDFA into a DFA **(6)**

δ :

	Present states	Input 0	Input 1
$\rightarrow$	q_0	q_0, q_2	q_1
	q_1	q_1	q_2
	q_2	q_0	q_1