ROLL NO. ___________________

Code: AE77/AC77/AE121 Subject: DIGITAL SIGNAL PROCESSING

AMIETE – ET/CS (Current & New Scheme)

Time: 3 Hours **December - 2017** Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The continuous time signal $x_c(t)=\sin(20\pi t)+\cos(40\pi t)$ is sampled with a sampling period T to obtain the discrete time signal $x_c(t)=\sin\left(\frac{\pi n}{5}\right)+\cos\left(\frac{2\pi n}{5}\right)$. What will be the choice of 'T' consistent with this information?

(A) T = 1/100 **(B)** T = 11/100

(C) Both **(A)** and **(B)** **(D)** None of these

b. Up sampler and down sampler are

(A) Time-varying system **(B)** Time-invariant system

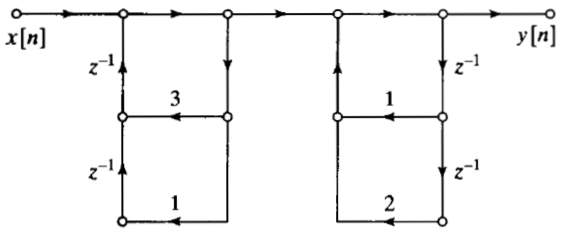
(C) Unpredictable system **(D)** May be time-varying or time-invariant system

c. Consider a linear time invariant system for which the input $x[n]$ and output $y[n]$ are related by a second-order difference equation $y[n-1]-\frac{10}{3}y[n]+y[n+1]=x[n]$. What are the locations of poles in the z-plane?

(A) Poles: 1/3 and 3 **(B)** Poles: -1/3 and -3

(C) Poles: 1/3 and -3 **(D)** None of these

d. A linear time-invariant system is realized by the flow graph shown below.



What is the system function?

(A) $\frac{1}{1+4z^{-1}+7z^{-3}+2z^{-4}}$ **(B)** $\frac{1}{1-4z^{-1}+7z^{-3}-2z^{-4}}$

(C) $\frac{1}{1-4z^{-1}-7z^{-3}+2z^{-4}}$ **(D)** None of these

e. For same set of specifications
   (A) IIR filter requires fewer filter coefficients than an FIR filter
   (B) FIR filter requires fewer filter coefficients than IIR filter
   (C) FIR and IIR require same number of filter coefficients
   (D) None of these

f. For Kaiser Window the width of main lobe is
   (A) $\frac{4\pi}{N}$ **(B)** $\frac{8\pi}{N}$
   (C) $\frac{12\pi}{N}$ **(D)** Adjustable

g. DFT of $\delta(n)$ is
   (A) 2π **(B)** π
   (C) 1 **(D)** 0

h. For the number of stages in the computation of DFT by radix-2 FFT to be 8. How many samples must $x(n)$ have?
   (A) 256 **(B)** 128
   (C) 18 **(D)** 8

i. A speech signal is sampled with a sampling rate of 16000 samples/sec (16 kHZ). A window of 20-ms duration is used in time dependant Fourier analysis of the signal. The window is advanced by 40 samples between the computations of the DFT. Assume that the length of each DFT is $N = 2^v$. How many samples are there in each segment of the speech selected by the window? How many DFT computations are done per second?
   (A) Length of Window = 320 samples and frame rate = 400 frames/sec
   (B) Length of Window = 400 samples and frame rate = 320 frames/sec
   (C) Length of Window = 300 samples and frame rate = 400 frames/sec
   (D) None of these

j. Consider a sequence x[n] and its discrete time Fourier transform $X\left(e^{j\omega}\right)$. The sequence x[n] is real valued and causal and $\mathrm{Re}\left\{X\left(e^{j\omega}\right)\right\} = 2 - 2a\cos\omega$. What is the value of $\mathrm{Im}\left\{X\left(e^{j\omega}\right)\right\}$?
   (A) $2a\cos\omega$ **(B)** $2a\sin\omega$
   (C) $-2a\sin\omega$ **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Consider the systems shown in figure given below. **(8)**

$$x[n] \rightarrow \boxed{\uparrow 2} \rightarrow \boxed{H_1\left(e^{j\omega}\right)} \rightarrow \boxed{\downarrow 2} \rightarrow y_1[n]$$

$$x[n] \rightarrow \boxed{H_2\left(e^{j\omega}\right)} \rightarrow y_2[n]$$

Suppose that $H_1\left(e^{j\omega}\right)$ is fixed and known. Find $H_2\left(e^{j\omega}\right)$ the frequency response of an LTI system such that $y_2[n] = y_1[n]$, if the inputs to the following system are same.

b. Consider the analog signal $x_a(t) = 3\cos 100\pi t$. Suppose the signal is sampled at the rate $F_{s1} = 75Hz$ and $F_{s2} = 200Hz$. What is the discrete time signal obtained after sampling? **(8)**

Q.3 a. A causal linear time-invariant system is described by the difference equation given below.

$$y[n-1] = \frac{3}{2}y[n-1] + y[n-2] + x[n-1]$$

Find the system function $H(Z) = Y(Z)/X(Z)$ for the system. Find the impulse response of the system. Find a stable (non-causal) impulse response that satisfies the difference equation. **(8)**

b. Let $x[n]$ be a causal N-point sequence that is zero outside the range $0 \le n \le N-1$. $x[n]$ is the input to the causal LTI system represented by the difference equation given below.

$$y[n] - \frac{1}{4}y[n-2] = x[n-2] - \frac{1}{4}x[n]$$

The output $y[n]$,is also causal, N-point sequence. Show that the causal LTI system described by this difference equation represents an all pass filter. Given that $\sum_{n=0}^{N-1}\left|x[n]\right|^2 = 5$, determine the value of $\left|y[n]\right|^2$. **(8)**

Q.4 a. Consider the LTI system with system function as given below.

$$H(z) = \frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$$

Draw the direct form-I and direct form-II structure. **(8)**

b. For many applications, it is useful to have a system that will generate a sinusoidal sequence. One possible way to do this is with a system whose impulse response is $h[n] = e^{j\omega_0 n}u[n]$. The real and imaginary part of $h[n]$ are therefore $h_r[n] = (\cos\omega_0 n)u[n]$ and $h_i[n] = (\sin\omega_0 n)u[n]$, respectively. Implement the system. **(8)**

Q.5 a. A low pass filter has to be designed with the following desired frequency response

$$H_d\left(e^{j\omega}\right)=\begin{cases} e^{-j2\omega} & -\dfrac{\pi}{4}\le\omega\le\dfrac{\pi}{4} \\ 0 & \dfrac{\pi}{4}<\left|\omega\right|\le\pi \end{cases}$$

Determine the filter coefficients $h_d(n)$ if the window function is defined as

$$w(n)=\begin{cases} 1 & 0<n\le 4 \\ 0 & otherwise \end{cases}$$

Also determine the frequency response $H\left(e^{j\omega}\right)$ of the designed filter. **(8)**

b. For the analog transfer function $H(s)=\dfrac{1}{(s+1)(s+2)}$, determine $H(z)$ using impulse invariant technique. Assume T=1s. **(8)**

Q.6 a. Show that with $x(n)$ as an N-point sequence, $X(k)$ is N-point DFT. Then prove that **(8)**

$$\sum_{n=0}^{N-1}\left|x(n)\right|^2=\frac{1}{N}\sum_{n=0}^{N-1}\left|X(K)\right|^2$$

b. Let $x(n)$ be a real valued sequence of length N, and let $X(k)$ be its DFT with real and imaginary parts $X_R(k)$ and $X_I(k)$ respectively. Show that, if $x(n)$ is real, $X_R(k)=X_R(N-k)$ and $X_I(k)=X_I(N-k)$ for k=1, 2,…., N-1. **(8)**

Q.7 An 8-point sequence is given by $x[n]=\{2,2,2,2,1,1,1,1\}$. Compute its 8-point DFT by

a. Radix-2 DIT FFT Algorithm **(8)**

b. Radix-2 DIF FFT Algorithm **(8)**

Q.8 a. Let x[n] be a discrete time signal whose spectrum you wish to estimate using a windowed DFT. You are required to obtain a frequency resolution of at least $\pi/25$ using a window length N=256. A safe estimate of the frequency resolution of a spectral estimate is the main-lobe width of the window used. Which of the window will satisfy the criteria given for frequency resolution? **(8)**

b. Let x[n] be a 5000 points sequence obtained by sampling a continuous time signal $x_c(t)$ at T = 50 μs. Suppose X[k] is the 8192 point DFT of x[n]. What is the equivalent frequency spacing in continuous time of adjacent DFT samples? **(8)**

Q.9 Let x[n] is a real, causal sequence with the imaginary part of its discrete time Fourier transform $X\left(e^{j\omega}\right)$ given by $\mathrm{Im}\left[X\left(e^{j\omega}\right)\right]=\sin\omega+2\sin 2\omega$. Is this answer unique? If so, explain why? If not, determine a second distinct choice for x[n] satisfying the relationship given above. **(16)**