ROLL NO. ___________________

Code: AE57/AC57/AT57/AE112 Subject: SIGNALS AND SYSTEMS

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **December - 2017** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Match the following:

Group 1	Group 2
a. Impulse	1. Sample values are unpredictable
b. Step	2. Has only one non-zero value
c. Random noise	3. Amplitude decreases and time increases
d. Decaying exponential	4. Has only two possible values

(A) a-3, b-2, c-4, d-1 **(B)** a-2, b-4, c-1, d-3
(C) a-1, b-2, c-3, d-4 **(D)** a-2, b-1, c-4, d-3

b. The period of a function $\cos\left(\frac{\pi}{4}(t-1)\right)$ is

(A) $\frac{1}{8}s$ **(B)** $8s$

(C) $4s$ **(D)** $\frac{1}{4}s$

c. Find the correct representation for the following:

(A) $x_1(n) * x_2(n) = x_2(n) * x_1(n)$

(B) $x_1(n) * [x_2(n) * x_3(n)] = [x_1(n) * x_2(n)] * x_3(n)$

(C) $x_1(n) * [x_2(n) + x_3(n)] = x_1(n) * x_2(n) + x_1(n) * x_3(n)$

(D) All are valid

d. Given the exponential form of Fourier Series $x(t) = \frac{A}{2} + j\frac{A}{2\pi}\sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{e^{jm\omega_0 t}}{m}$ trigonometric form is

(A) $\frac{A}{2} + \frac{A}{\pi}\sum_{m=1}^{\infty}\frac{\sin(m\omega_0 t)}{m} \quad \forall m$ (B) $\frac{A}{2} - \frac{A}{\pi}\sum_{m=1}^{\infty}\frac{\sin(m\omega_0 t)}{m} \quad \forall m$

(C) $\frac{A}{2} - \frac{A}{2\pi}\sum_{m=1}^{\infty}\frac{\sin(m\omega_0 t)}{m} \quad \forall m$ (D) $\frac{A}{2} - \frac{A}{\pi}\sum_{m=1}^{\infty}\frac{\cos(m\omega_0 t)}{m} \quad \forall m$

e. What name is given to the lowest frequency in the Fourier Series

(A) Fundamental **(B)** First harmonic
(C) Both **(A)** and **(B)** **(D)** None of these

f. If the Fourier transform of $x(t)$ is $X(\omega)=\frac{2}{1+\omega^2}$, then the Fourier transform $x(t)e^{-jat}$ is

(A) $\frac{2}{1+a^2}$ **(B)** $\frac{2}{1-a^2}$
(C) $\frac{2}{1+(\omega+a)^2}$ **(D)** $\frac{2}{1+(\omega-a)^2}$

g. The Fourier transform of $x(t)=e^{-t}u(t)$ is $\frac{1}{1+j\omega}$, then the transformation $y(t)=x(t)-x(-t)$ is

(A) $\frac{1}{1-\omega^2}$ **(B)** $\frac{\omega}{1+\omega^2}$
(C) $\frac{-j\omega}{1-\omega^2}$ **(D)** $\frac{-j2\omega}{1+\omega^2}$

h. Three sine waves with frequencies 100 Hz, 200 Hz and 350 Hz having amplitudes 1 V, 2 V and 1.5 V respectively are added together to form a single waveform. What is the minimum sampling frequency, out of the following, that will enable the satisfactory reconstruction of the original signal?

(A) 750 Hz **(B)** 650 Hz
(C) 1050 Hz **(D)** 400 Hz

i. The z-transform of $e^{2n}u(n)$ is

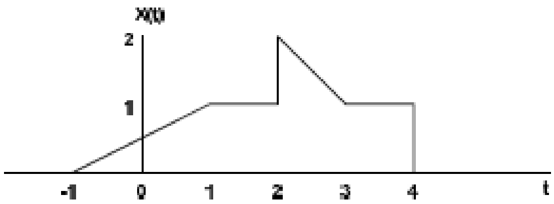
(A) $\frac{1}{1-e^2z^{-1}}, ROC:|z|<|e^2|$ **(B)** $\frac{1}{1+e^2z^{-1}}, ROC:|z|<|e^2|$
(C) $\frac{1}{1-e^2z^{-1}}, ROC:|z|>|e^2|$ **(D)** $\frac{1}{1+e^2z^{-1}}, ROC:|z|>|e^2|$

j. If $\delta(t)$ denotes an unit impulse, then Laplace transform of $\frac{d^2\delta(t)}{dt^2}$ will be

(A) 1 **(B)** s^2
(C) s **(D)** s^{-2}

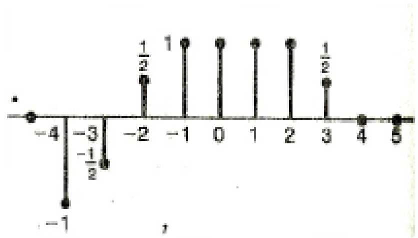
Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Represent the signal x(t) shown below as a linear combination of standard test signals **(2)**



b. If I/O relationship of discrete time system is $y(n) = \begin{cases} x(n) & n \geq 1 \\ 0 & n = 0 \\ x(n+1) & n \leq -1 \end{cases}$. Check for the following properties of the given system i) Memory less or with memory ii) Causality **(2)**

c. Sketch and label $x[n-4]$, $x[3-n]$ and $x[3n+1]$ for the signal $x[n]$ shown below:

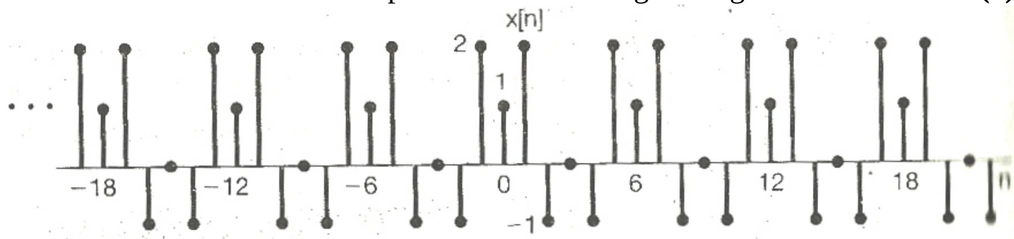


(4)

d. If $x(t) = u(t) - u(t-1)$ and $h(t) = u(t) - u(t-3)$ Compute $y(t) = x(t) * h(t)$ **(8)**

Q.3 a. What do you understand by Gibb's phenomenon. **(4)**

b. Determine the Fourier series representation for the given signal. **(4)**

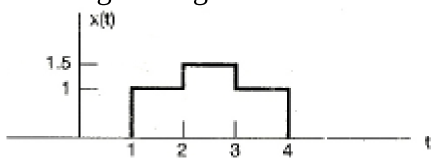


c. If $x(n) = 1 + \cos(\frac{2\pi}{6}n)$, $y(n) = \sin(\frac{2\pi}{6}n + \frac{\pi}{4})$ and $z(n) = x(n)y(n)$. Calculate the Fourier series coeffs. of z(n)? **(8)**

Q.4 a. Prove that Fourier Transform maps the convolution of two signals into the product of their Fourier Transforms. Use this property to obtain the input $x(t)$ for causal LTI system output $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$ and frequency response $H(j\omega) = \dfrac{1}{j\omega + 3}$. **(8)**

b. State and prove Duality property by taking an example. **(4)**

c. Obtain Fourier transform of the given signal. **(4)**



Q.5 a. Determine the Fourier transform for $-\pi \leq \omega < \pi$ for the following periodic signal: $2 + \cos\left(\frac{\pi}{6}n + \frac{\pi}{8}\right)$ **(6)**

b. Given that $x(n)$ has Fourier transform $X(e^{j\omega})$, express the Fourier transforms of the following signals in terms of $X(e^{j\omega})$. Use Fourier transform properties:
(i) $x_1(n) = x(1-n) + x(-1-n)$ (ii) $x_2(n) = \dfrac{x^*(-n) + x(n)}{2}$ (iii) $x_3(n) = (n-1)^2 x(n)$ **(10)**

Q.6 a. Discuss various issues in magnitude-phase representation of frequency response of a LTI system. **(8)**

b. State sampling theorem. Explain, how will you recover the original signal after Sampling? **(8)**

Q.7 a. A pressure gauge that can be modelled as LTI system has a time response to a unit step input given by $(1 - e^{-t} - te^{-t})u(t)$. For a certain input $x(t)$ the output is observed to be $(2 - 3e^{-t} + e^{-3t})u(t)$. For this observed measurement determine the true pressure input to the gauge as a function of time **(10)**

b. Consider a LTI system with system function $H(s) = \dfrac{(s-1)}{(s+1)(s-2)}$. Find its inverse transform if (i) system is causal (ii) system is stable **(6)**

Q.8 a. For a signal $x[n]$ with z-transform $X(z)$, following five facts are given;
(i) $x[n]$ is real and right sided.
(ii) $X(z)$ has exactly two poles.
(iii) $X(z)$ has two zeros at the origin.
(iv) $X(z)$ has a pole at $z = \frac{1}{2}e^{j\frac{\pi}{3}}$
(v) $X(1) = \frac{8}{3}$.
Determine $X(z)$ and specify its ROC. **(10)**

b. Find $X(z)$ and its ROC for $x[n] = \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4}n\right)u[n]$. Indicate if Fourier Transform exists for $x[n]$. **(6)**

Q.9 a. Describe with suitable derivations transmission of a random process through a linear Time-invariant filter. **(8)**

b. Consider a white Gaussian noise process of zero mean and power spectral density $N_0/2$ that is applied to the input of the high-pass RL filter shown in Fig. below.
(i) find the autocorrelation function and power spectral density of the random process at the output of the filter (ii) What are the mean and variance of this output? **(8)**