

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours

December - 2017

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of the integral $\int_c \frac{c^{iz}}{z+3i} dz$, where c is the circle $|z+3i|=1$, measured

in the counter clockwise sense is

- (A) $2\pi i$ (B) 0
(C) $2\pi ic^{-3}$ (D) $2\pi ic^3$

b. The residue of $\frac{1}{(z^2+a^2)^2}$ at $z=ai$ is

- (A) $\frac{i}{4a^3}$ (B) $-\frac{i}{4a^3}$
(C) $\frac{i}{a^3}$ (D) $\frac{2i}{a^3}$

c. The value of λ and μ by using vectors, such that the points $(-1, 3, 2)$, $(-4, 2, -2)$ and $(5, \lambda, \mu)$ lie on a straight line are

- (A) $\lambda=2, \mu=3$ (B) $\lambda=5, \mu=1$
(C) $\lambda=5, \mu=10$ (D) $\lambda=4, \mu=6$

d. The values of $\int_c (x^2+xy)dx \cdot (x^2+y^2)dy$ where c is the square formed by the lines is

- (A) 1 (B) 0
(C) 2 (D) 5

e. If $f(x) = 2x^3 - 3x^2 + 3x - 10$, then $\Delta^3 f(x)$ is equal to

- (A) 10 (B) 15
(C) 0 (D) 12

- f. If $f(3) = 24$, $f(5) = 120$, $f(8) = 504$, $f(9) = 720$ and $f(12) = 1716$. Use Newton's divided difference formula to find $f(7)$
 (A) 335 (B) 336
 (C) 337 (D) 338
- g. Eliminating a , b from $z = (x+a)(y+b)$, we obtain the partial differential equation
 (A) $z = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$ (B) $z = \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)$
 (C) $z = 1 + \left(\frac{\partial z}{\partial x}\right)^2$ (D) $z = x + \left(\frac{\partial z}{\partial y}\right)^2$
- h. The probability of throwing 9 with two dices is
 (A) 9/16 (B) 13/18
 (C) 1/9 (D) 3/4
- i. The probability of getting 4 heads in 6 tosses of a fair coin is
 (A) 1/32 (B) 5/16
 (C) 15/64 (D) 3/8
- j. What is the variance of the binomial distribution?
 (A) npq (B) \sqrt{npq}
 (C) np (D) None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

- Q.2** a. Test the analyticity of the function $w = \sin z$ and hence derive that
 $\frac{d}{dz}(\sin z) = \cos z$ (8)
- b. Find the mapping of x-axis under the transformation $w = \frac{i-z}{i+z}$, onto the w -plane. (8)
- Q.3** a. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$. (8)
- b. Determine the poles of the following function and residue at each pole:
 $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\int_c \frac{z^2 dz}{(z-1)^2(z+2)}$ where $c: |z|=3$. (8)
- Q.4** a. Show that $\vec{v}(x, y, z) = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + x^2y\hat{k}$ is irrotational and find a scalar function $u(x, y, z)$ such that $\vec{v} = \text{grad}(u)$. (8)

- b. Find the directional derivative of $f(x, y, z) = x^2y^2z^2$ at the point (1, 1, -1) in the direction of the tangent to the curve $x = e^t$, $y = 2\sin t + 1$, $z = 1 - \cos t$ at $t=0$. (8)

- Q.5** a. If $\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$, evaluate $\iiint_v \vec{F}dv$ where v is the region bounded by the surfaces $x=0$, $y=0$, $x=2$, $y=4$, $z=x^2$, $z=2$. (8)

- b. Verify Stoke's theorem for the function $f = x2\hat{i} - xy\hat{j}$ integrated round the square in the plane $z=0$ and bounded by the lines $x=0$, $y=0$, $x=a$, $y=a$. (8)

- Q.6** a. Solve $(x^2 - y^2)p + (y^2 - 2x)q = z^2 - xy$ (8)

- b. Solve the partial differential equation $(p^2 + q^2)y = qz$ by Charpit's method. (8)

- Q.7** a. Find the cubic polynomial, which takes the following values (8)

x	0	1	2	3
$f(x)$	1	2	1	10

Hence, evaluate $f(4)$.

- b. The velocity (v) of a car which starts from rest, is given at fixed intervals of time (t) as follows: (8)

t	2	4	6	8	10	12	14	16	18	20
v	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered by car in 20 units of time.

- Q.8** a. A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $1/7$ and that of wife selection is $1/5$. What is the probability that (8)

- (i) both of them will be selected
(ii) only one of them will be selected
(iii) none of them will be selected

- b. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. If their output 5, 4 and 2 percents are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B? (8)

- Q.9** a. The overall percentage of failures in a certain examination is 20. If six candidates appear in the examination, what is the probability that at least five pass the examination? (8)

- b. The diameter of an electric cable is assumed to be continuous random variable with probability density function:

$$f(x) = 6x(1-x), \quad 0 \leq x \leq 1$$

- (i) Verify that above is a probability density function.
(ii) Find the mean and variance. (8)