ROLL NO. ____________________

Code: AE51/AC51/AT51/AE101/AC101/AT101
Subject: ENGINEERING MATHEMATICS - I

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours | **December - 2017** | **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 to Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTIONS FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

b. If $z = \sin^{-1}(x^2 + y^2)$,then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(A) $\tan z$ **(B)** $\cot z$
(C) $2\tan z$ **(D)** $2\cot z$

c. The value of $\int_0^1\int_0^x xdydx$ is

(A) 3/2 **(B)** 1
(C) 2/3 **(D)** 1/3

d. The approximation to a root of the equation $f(x) = 0$ by Regula-falsi method is

(A) $x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$ **(B)** $x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_1) - f(x_0)}$

(C) $x_2 = \frac{x_0 f(x_0) - x_1 f(x_1)}{f(x_0) - f(x_1)}$ **(D)** $x_2 = \frac{x_1 f(x_1) - x_0 f(x_0)}{f(x_0) - f(x_1)}$

e. The solution of the differential equation $(3x^2 - 2xy)dx = x^2 dy$ is

(A) $x^3 - xy = c$ **(B)** $x - x^3 y = c$
(C) $x^3 - x^2 y = c$ **(D)** $x^3 - x^2 y^2 = c$

f. The Wronskian of x and e^x is

(A) 1 **(B)** e^x
(C) $x - e^x$ **(D)** $(x-1)e^x$

g. The particular integral of $y'' - 4y' = \sin 2x$ is

(A) $\frac{-\sin 2x}{8}$ **(B)** $\frac{-\cos 2x}{8}$
(C) $\frac{-\sin 2x}{2}$ **(D)** $\frac{-\cos 2x}{2}$

h. $\beta(m, n+1) + \beta(m+1, n) =$

(A) $\beta(m+1, n+1)$ **(B)** $\beta(m, n)$
(C) $\beta(2m+1, n)$ **(D)** $\beta(m, 2n+1)$.

i. The value of $P_n(-1)$ is

(A) n **(B)** - n
(C) $(-1)^n$ **(D)** None of these

j. The value of $J_2(x)$ in terms of $J_0(x)$ and $J_1(x)$ is

(A) $\frac{2}{x} J_1(x) - J_0(x)$ **(B)** $\frac{4}{x} J_1(x) - J_0(x)$
(C) $\frac{2}{x} J_0(x) - J_1(x)$ **(D)** $\frac{4}{x} J_0(x) - J_1(x)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$. **(8)**

b. Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. **(8)**

Q.3 a. Evaluate $\int_0^1 \int_0^{\sqrt{2x - x^2}} \frac{xdydx}{\sqrt{x^2 + y^2}}$ by changing to polar co-ordinates **(8)**

b. Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ **(8)**

Q.4 a. Test for consistency and solve by Gauss elimination method the system of equations: $x+4y-z=-5, x+y-6z=-12$, $3x-y-z=4$ **(8)**

b. Find the Eigen values and the corresponding Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(8)**

Q.5 a. Apply Newton-Raphson method to find a real root of the equation $3x=\cos x+1$ correct to three decimal places. Take $x_0=0.5$. **(8)**

b. Solve the system of equation
$8x-3y+2z=20, 4x+11y-z=33, 6x+3y+12z=36$
using Gauss-Seidel method. Carry out four iterations. **(8)**

Q.6 a. Solve $(D^2+4)y=\tan 2x$ by method of variation of parameters **(8)**

b. Solve $x\frac{dy}{dx}+y=x^3y^6$. **(8)**

Q.7 a. Obtain the series solution of $\frac{d^2y}{dx^2}+x^2y=0$. **(8)**

b. Prove that $\beta(m,n)=\frac{\Gamma m\ \Gamma n}{\Gamma(m+n)}$. **(8)**

Q.8 a. Prove that $\int_{-1}^{1} x^2 P_{n+1}(x)P_{n-1}(x)dx=\frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$. **(8)**

b. Show that $x^2J_n''(x)=(n^2-n-x^2)J_n(x)+xJ_{n+1}(x)$. **(8)**

Q.9 **(For Current Scheme students. i.e. AE51/AC51/AT51)**

a. Solve $(xy^2-e^{1/x^3})dx=x^2ydy$. **(8)**

b. Form the differential equation of all circles of radius ‘r’ units **(8)**

Q.9 **(For New Scheme students. i.e. AE101/AC101/AT101)**

a. Find the Fourier cosine transform of e^{-x^2}. **(8)**

b. Use convolution theorem to evaluate $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$ **(8)**